MAT132, Paper Homework 5

due in recitation on 10/17, 10/18, or 10/19

1. According to Newton's law of gravitation, two bodies with masses m_1 and m_2 attract each other with a force

$$F = G\frac{m_1 m_2}{r^2},$$

where r is the distance between the bodies, and $G = 6.67 \times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2$ is the gravitational constant.

   (a) Recall that, near the surface of the earth, the gravitational force on an object of mass m is given by $F = mg$, where $g = 9.81\mathrm{m}/\mathrm{s}^2$. Use this fact to compute the mass of the earth! Cavendish, who in 1798 was the first to measure G, called his experiment "weighing the earth" for that reason. (Hint: The radius of the earth is $6.37 \times 10^6\mathrm{m}$.)

   (b) Find the escape velocity v_0 needed to propel a rocket of mass m out of the gravitational field of a planet with mass M and radius R. (Hint: The initial kinetic energy of the rocket, equal to $\frac{1}{2}mv_0^2$, provides the needed work.)

2. The *standard deviation* for a random variable X with probability density function f and mean μ is defined by the formula

$$\sigma = \left[\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx\right]^{\frac{1}{2}}.$$

It measures how far the values of X are away from the mean. Find the standard deviation for an exponential density function with mean μ.