MAT132, Paper Homework 3
due in recitation on 9/26, 9/27, or 9/28

1. This problem asks you to show that

$$\int \sec x \, dx = \ln\left|\sec x + \tan x\right| + C. \tag{1}$$

   (a) Write $\sec x$ as $\cos x/(1 - \sin^2 x)$ and make a substitution to simplify the integral.

   (b) Integrate the resulting rational function by the partial fractions method.

   (c) Use the identity $\sin^2 x + \cos^2 x = 1$ to show that your result is equal to (1).

   Note: In case you are interested, here is another clever method:

$$\int \sec x \, dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx = \ln\left|\sec x + \tan x\right| + C$$

   because the numerator is the derivative of the denominator.

2. We let S be the region defined by

$$S = \left\{ (x, y) \,\middle|\, -2 < x \le 0 \text{ and } 0 \le y \le 1/\sqrt{x+2} \right\}.$$

   (a) Sketch the region. (*Please make a sufficiently large sketch.*)

   (b) Write the area of S as an improper integral and find its value.