MAT132, Paper Homework 1
due in recitation on 9/12, 9/13, or 9/14

1. Let
$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } 1 < x \geq 2 \\ 0 & \text{if } x > 2 \end{cases}$$
and define a new function $g(x) = \int_0^x f(t)\, dt$.
   (a) Find an expression for $g(x)$ similar to the one for $f(x)$.
   (b) Sketch the graphs of f and g.

2. In the problem below, the identities $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ and $\sin^2(x) + \cos^2(x) = 1$ will be useful.
   (a) Use substitution to show that $\int_0^{\pi/2} f(\sin x)\, dx = \int_0^{\pi/2} f(\cos x)\, dx$ for *any* continuous function f.
   (b) Using part (a) and the second identity above, calculate $\int_0^{\pi/2} \sin^2(x) dx$ and $\int_0^{\pi/2} \cos^2(x) dx$.