MATH 132 Solutions to Practice 2

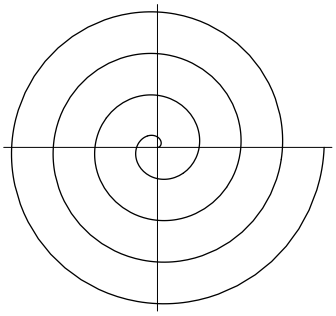
1234 pts

1. There are a bunch of problems about work and volume and stuff on the Spring 2010 exam. So those aren't here. Go do those, OK? If you already did them, here are the solutions.

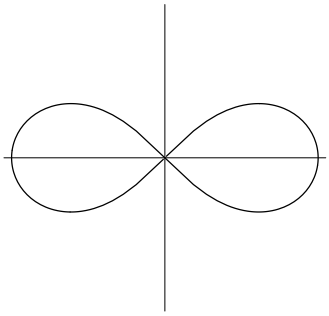
47 pts

2. Match the following polar equations to their graphs. Please write the letter of the graph in the space preceeding the equation. Note that, although each graph is accurate, two different graphs may be drawn at different scales.

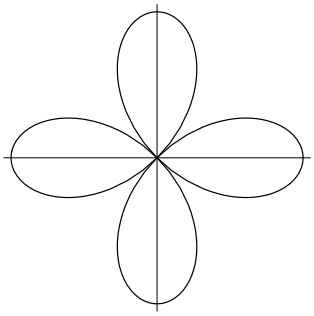
C $r = \cos 2\theta$ **A** $r = \theta \ \ (\theta > 0)$ **D** $\theta = \frac{\pi}{4}$ **E** $r = 1 + 2\sin\theta$



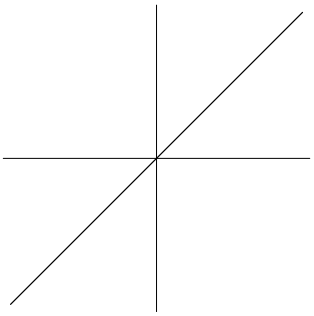
A



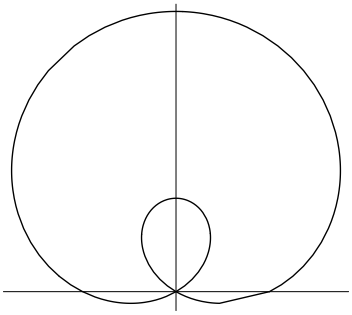
B



C



D



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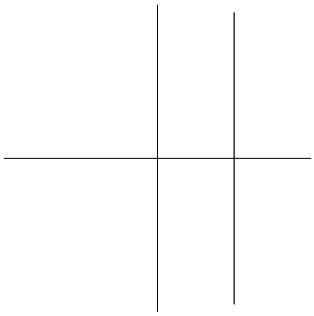
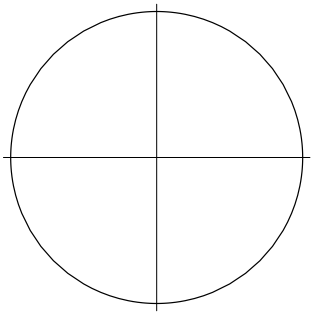
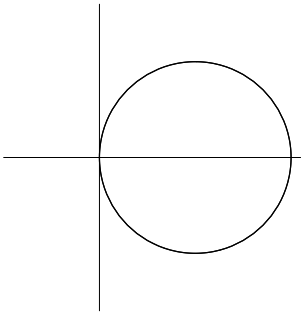


F

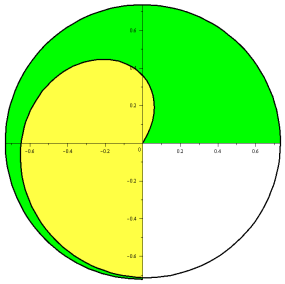


G



H

47 pts 3. At right is shown the graph of the polar curve



$$r = \frac{\ln\theta}{\sqrt{\theta}} \qquad 1 \le \theta \le \frac{7\pi}{2}$$

Calculate the area of the green region.

Solution: Note that for $1 \le \theta \le \frac{3\pi}{2}$, the curve $r = \frac{\ln\theta}{\sqrt{\theta}}$ traces out the inner curve, so $\frac{1}{2}\int_1^{3\pi/2} \left(\frac{\ln\theta}{\sqrt{\theta}}\right)^2 d\theta$ represents the yellow area.

Similarly, the integral $\frac{1}{2}\int_{2\pi}^{7\pi/2} \left(\frac{\ln\theta}{\sqrt{\theta}}\right)^2 d\theta$ represents the green and yellow areas.

This means that the green area will be given by their difference, i.e.

$$\frac{1}{2}\int_{2\pi}^{7\pi/2} \frac{\ln^2\theta}{\theta}\,d\theta - \frac{1}{2}\int_1^{3\pi/2} \frac{\ln^2\theta}{\theta}\,d\theta.$$

Let $u = \ln\theta$, so $du = 1/\theta\,d\theta$. We also adjust the limits of integration and get

$$\begin{aligned}\frac{1}{2}\int_{\ln(2\pi)}^{\ln(7\pi/2)} u^2\,du - \frac{1}{2}\int_0^{\ln(3\pi/2)} u^2\,du &= \frac{1}{2}\left(\frac{u^3}{3}\Big|_{\ln(2\pi)}^{\ln(7\pi/2)} - \frac{u^3}{3}\Big|_0^{\ln(3\pi/2)}\right) \\ &= \frac{1}{6}\left((\ln(7\pi/2))^3 - (\ln(2\pi))^3 - (\ln(3\pi/2))^3\right)\end{aligned}$$