Problem 1. Let $h(t) = \dfrac{\sin^3(\pi t) + e^{\sqrt{t}}}{\arctan(1-t^2) - t}$ and let $G(x) = \int_0^x h(t)\,dt$.

(a) Find $\int_{-1}^{1} h'(t)\,dt$.

(b) Find $G'(0)$.

Problem 2. Determine whether the following integrals converge or diverge. Explain your answer completely. Find the exact answer if possible.

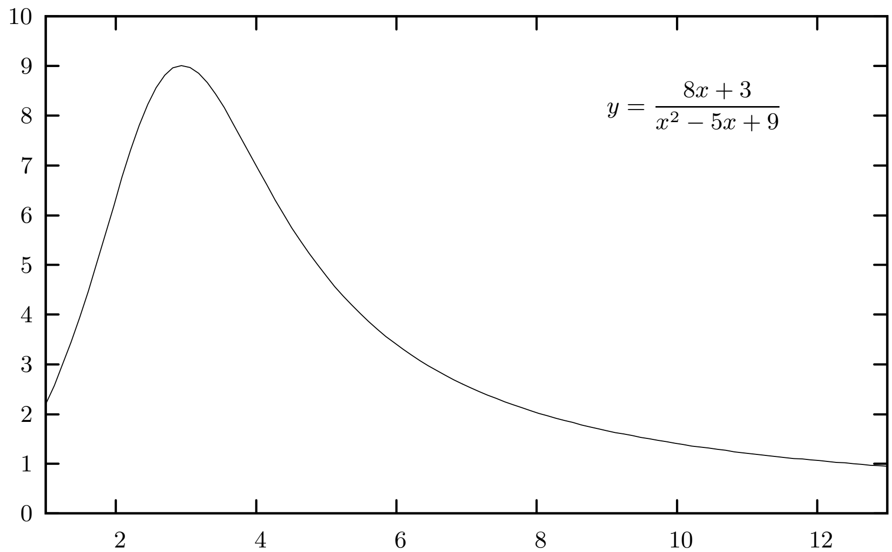
(a) $\displaystyle\int_2^{\infty} \frac{dx}{x\ln(x)}$

(b) $\displaystyle\int_0^{\infty} \frac{dx}{1+x^3}$

(c) $\displaystyle\int_0^{\infty} \frac{x}{e^x}\,dx$

(d) $\displaystyle\int_0^{2} \frac{dx}{\sqrt{2-x}}$

Problem 3.



(a) Use the left hand rule with $n = 5$ to approximate $\int_2^{12} \frac{8x+3}{x^2-5x+9} dx$.

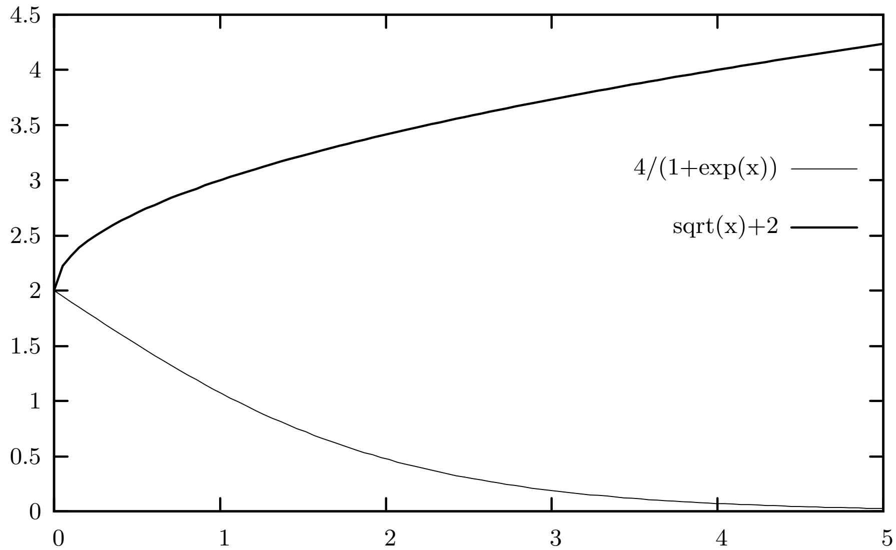
(b) Use the right hand rule with $n = 5$ to approximate $\int_2^{12} \frac{8x+3}{x^2-5x+9} dx$.

(c) Use the trapezoid hand rule with $n = 5$ to approximate $\int_2^{12} \frac{8x+3}{x^2-5x+9} dx$.

(d) Use the midpoint rule with $n = 5$ to approximate $\int_2^{12} \frac{8x+3}{x^2-5x+9} dx$.

(e) Use Simpson's rule with $n = 4$ to approximate $\int_2^{10} \frac{8x+3}{x^2-5x+9} dx$.

Problem 4. **[20 points]** Consider the region trapped by the two curves $y = \dfrac{4}{1+e^x}$ and $y = \sqrt{x} + 2$ and between the lines $x = 0$ and $x = 5$. Here is a picture of the region:



(a) Use an integral to express the volume of the solid formed by rotating this region around the x-axis. Do not evaluate the integral.

(b) Use an integral to express the volume of the solid formed by rotating this region around the line $x = 5$. Do not evaluate the integral.

EXAM

Practice Midterm 1

Math 132

February 20, 2004