# PAPER HOMEWORK 11

MAT 127, SPRING 2018

**1.** Let $k$ be a real number. Use the ratio test to show that the binomial series

$$\sum_{n=0}^{\infty} \binom{k}{n} x^n = \sum_{n=0}^{\infty} \frac{k(k-1)\cdots(k-n+1)}{n!} \cdot x^n$$

converges for $-1 < x < 1$.

**2.** The goal of this exercise is to prove that

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

for $-1 < x < 1$.

(a) Define a function

$$B(x) = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

for $-1 < x < 1$. Differentiate the binomial series to show that

$$B'(x) = \frac{kB(x)}{1+x}.$$

(b) Let $C(x) = (1+x)^{-k} B(x)$. Use (a) to show that $C'(x) = 0$.

(c) Deduce that $B(x) = (1+x)^k$ for $-1 < x < 1$.

**3.** Find the Taylor series of the function $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$.

*Date*: Due by Wednesday, April 18.