Final exam
MAT 127

________________ , ________________
Last Name , First Name

________________ ________
I.D.# Lecture#

Question	Points	Score
1	10	
2	20	
3	30	
4	50	
5	20	
6	40	
7	10	
8	20	
Total:	200	

Stop!
Do Not Open This Exam Booklet
Until You Are Told to Do So!

Exam Rules:

No Calculators. No Books. No Notes.

Please show all your work, explain your reasoning, and cross out anything we should ignore when grading this exam. Also where possible, please always give exact answers (for example, "$\sqrt{5}$" rather than the decimal approximation "2.23"). Good luck!

You have 150 minutes to complete this exam.

There are 8 questions, for a total of 200 points. Good luck!

Name: ______________________ **Id:** ______________________

1. (10 points) Find the solution $y(x)$ of the intial value problem

$$\begin{aligned} \frac{dy}{dx} &= y^2 \sin x \\ y(0) &= 1. \end{aligned}$$

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2. After 3 days a sample of radon-222 decayed to 60% of its original amount.

   (a) (10 points) What is the half-life of radon-222?

   (b) (10 points) How long would it take the sample to decay to 10% of its original amount?

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3. Determine whether the following sequences are convergent or not. If convergent compute their limits. Show your work!

   (a) (10 points)

$$a_n = \frac{9^{n+1}}{10^n}$$

   (b) (10 points)

$$a_n = \frac{n^3}{n^2+1}$$

   (c) (10 points)

$$a_n = \ln(n+2) - \ln(n+1)$$

4. (a) (10 points) Does the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} = \frac{1}{2(\ln 2)^2} + \frac{1}{3(\ln 3)^2} + \cdots$$

converge or diverge? Explain why.

(b) (15 points) Does

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1} = \frac{1}{2} - \frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{4} - \frac{2}{5} + \cdots$$

converge or diverge? Is the series absolutely convergent? Explain why.

(c) (10 points) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{2^{2n+1}}{5^n}$$

Please show your work and explain your reasoning!

(d) (15 points) Does the series

$$\sum_{n=1}^{\infty} \frac{5^{2n}}{n^2 9^n}$$

converge of diverge? Explain why.

5. (20 points) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n}{2^n}(x-1)^n$$

Please show your work and explain your reasoning!

6. (a) (20 points) Expand the function $f(x) = \frac{x}{(1-x)^2}$ as a power series nearby $x = 0$; in other words compute the MacLaurin series for $f(x)$. What is its radius of convergence?

(b) (20 points) Compute the sum of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$$

How many terms of the series does one need to add/substract in order to compute the sum correct to one decimal place?

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7. (10 points) Use Taylor/MacLaurin series to evaluate the following limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

8. (a) (15 points) Use power series to solve the initial value problem

$$y' = x^2 y, \qquad y(0) = 1$$

(b) (5 points) What is the radius of convergence of the solution? Can you express the power series solution in terms of known elementary functions?