# MAT303: Calc IV with applications

Lecture 1 - February 03 2021

MAT303: Differential Equations

$P = 3$

$\frac{dP}{dt} = 0$

Today:

- What is a differential equation
- Why we should study differential equations
- Ch1.1: Differential equations and mathematical models
- Ch1.2: Integrals as solutions to differential equations

What/why:

- Many processes in the world can be described by their rate of change
- Rate of change <-> derivative
- Equations involving derivatives are *differential equations*.
- Differential equations allow us to study mathematical models of physical processes.

$P$

$1 = \frac{1}{2}kP \Rightarrow P = \frac{2}{k}$

Then $\frac{dP}{dt} = 0$

$y = 3x + 1$

$\frac{dy}{dx} = 3$

Example:

- Population grows at a rate proportional to the current population
- Derivative of population is proportional to the current population
- $\frac{dP}{dt} = kP$ <- Looking for function whose derivative is k times itself
- Solution is ?

current time.

a) $P = \frac{1}{2}kP^2$

b) $P(t) = \frac{1}{2}kt^2$

$\frac{dP}{dt} = kt$

c) $P(t) = e^{kt}$

d) $P(t) = 2e^{kt}$

$\frac{dP}{dt} = 2ke^{kt} = k\,2e^{kt}$

The problem is that it is easy to describe,
but hard to answer questions like ‘what is the population at time 10’?

Analogy:

- Some "real world" problem
- .
- $x^2 - 3x + 2 = 0$
- Solutions are $x = 2$ and $x = 1$

$2^2 - 3 \cdot 2 + 2$
$= 4 - 6 + 2 = 0$.

$1^2 - 3 \cdot 1 + 2 = 0$ ✓

$x^3 + 3x^2 + 3x + 1 = 0$

Similarities:

- There can be multiple solutions
- There can be no solutions
- It is easier to verify a solution that it is to find it.

$x^2 + 1 = 0$

Differences:

- For an algebraic equation, we are looking for a **number** to solve the equation
- For a differential equation, we are looking for a **function** to solve the equation

Example:

- Population grows at a rate proportional to the current population
- Derivative of population is proportional to the current population
- $\frac{dP}{dt} = kP$
- Solution is ~~$P(t) = Ce^{kt}$, where $C$ is any constant.~~

$P(t) = 2e^{kt}$

$P(t) = e^{kt}$

$ke^{kt} = ke^{kt}$ ✓

- How to interpret a differential equation
  - How to connect the differential equation to what you are modeling
  - How to go from a model to a differential equation
- Some techniques for solving the most basic and common differential equations
  - See how the setup is reflected in the solution

Ideally, after this class,

- You will be able to recognize the most basic and common DEs
- You will recognize when it is suitable to model something as a DE
- You will not be surprised when DEs come up in your work

Main resource: course website

http://www.math.stonybrook.edu/~bplin/teaching/spring2021/mat303/index.html

Syllabus, Lectures notes, schedule, hw, etc.
It is your responsibility to check this regularly.

entry.

Two services to sign up for. Links are on course website.

- Piazza: discussion board and announcements
- Gradescope: you must submit your homework here

Assessment (see syllabus)

- Homework (30%): due most Wednesdays. First one due Feb 10.
- Midterms (40%): March 3 and April 7 in class.
- Final (30%): May 18 5:30pm-8pm.

$T = A + Ce^{-kt}$

**Newton's law of cooling:**
The rate of change of the temperature T of a body is proportional to the difference between between T and the temperature A of the surrounding medium.

$T - A = \cancel{A} + Ce^{kt} \cancel{- A}$

A

t

Should k be
a) positive
b) negative.
c) I don't know.

Differential equation:

$$\frac{dT}{dt} = k(T - A)$$

rate of change of (T)emperature.

Notice how this is much more compact and clearer than the description in words.

What is temp at time 10?

$T(10) = ?$

Solution: For any C,

$T = A + Ce^{+kt}$ is a solution

A,C,k are constants. C is a new constant. Can be anything.

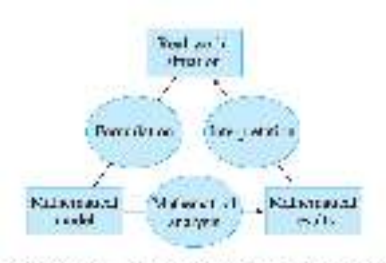
Verify: Substitute into $\frac{dT}{dt} = k(T-A)$

$$\frac{dT}{dt} = 0 + kCe^{kt}$$

$$k(T - A) = kCe^{kt}$$

$T = A + Ce^{kt}$

$T(10)$
$= A + Ce^{k \cdot 10}$



• → 3 mi/h east.

Where is the boat at 10 pm August 12 2021?

In the previous example,

$$\frac{dT}{dt} = k(T - A)$$

$$T = A + Ce^{kt}$$

$$T(t) = A + Ce^{kt}$$

There were infinitely many solutions, depending on the constant C.
How do we choose the correct solution to correspond to our physical model?

Answer: choose the correct initial conditions.

T(0)=A+C, so C = T(0)-A.

$T(0) = A + C$ so $C = A - T(0)$

so

$$T(t) = A + (A - \overset{100^\circ F}{T(0)})e^{kt}$$

General Principle:

Differential eqns have infinitely many solns.

if we add an initial condition, the soln becomes unique.

In the previous example,

$$\frac{dT}{dt} = k(T - A)$$

$$T = A + Ce^{-kt}$$

There were infinitely many solutions, depending on the constant C.
How do we choose the correct solution to correspond to our physical model?

Answer: choose the correct initial conditions.

T(0)=A+C, so C = T(0)-A.

Another example: verify that $y(x) = 2x^{1/2} - x^{1/2}\ln x$

Is solution to the differential equation with initial value

$$4x^2y'' + y = 0$$
$$y(0) = 0$$

$y(0) = \lim_{x\to 0} x^{1/2}\ln x = 0.$

$$y' = x^{-1/2} - \left(\frac{1}{2}x^{-1/2}\ln x + x^{-1/2}\right)$$

$$= -\frac{1}{2}x^{-1/2}\ln x$$

$$y'' = \frac{1}{4}x^{-3/2}\ln x - \frac{1}{2}x^{-3/2}$$

$$4x^2y'' + y$$

$$= x^{1/2}\ln x - 2x^{1/2} + 2x^{1/2} - x^{1/2}\ln x$$

$$= 0.$$

Simplest type of differential equation:

$$\frac{dy}{dx} = f(x)$$

$\int x^2 dx = \frac{1}{3}x^3$

Example:

$$\frac{dy}{dx} = 2x + 3, \quad y(1) = 2$$

So $y = \int 2x+3 \, dx$

$= x^2+3x+C$

Want

$y(1)=2$

So $4+C=2$

$C=-2$.

So the solution is

$y = x^2+3x-2$.

Verify:

$\frac{dy}{dx} = 2x+3$

$y(1) = 2$.

Very different:

$\frac{dy}{dx} = 2y$ soln is $Ce^{2x}$.

Simplest type of differential equation:

$$\frac{dy}{dx} = f(x)$$

$$y = \int f(x)\,dx$$

$$=$$