MAT303: Calc IV with applications

Lecture 8 - March 1 2021

Recently:

Ch 2.1 population models

$$\frac{dP}{dt} = (\beta - \delta)P$$

Ch 2.2 Analysis of $\dfrac{dP}{dt} = f(P)$

- Equlibria
- Stability
- Bifurcation

Today:

Ch 2.3 Acceleration/Velocity Models

$$\frac{dv}{dt} = F$$

Question:

- How do assumptions about air resistance affect the terminal velocity?

Question:

- Suppose a lunar lander is falling towards the moon.
- We want the velocity at landing to be 0.
- When should we turn on the upwards thrusters?

Question:

- How fast do we have to launch from the ground to escape the earth's orbit?

Newton's second law:

$$\frac{dv}{dt} = F$$

Today we will only consider:

$$\frac{dv}{dt} = \text{External Force} + \text{Gravity} + \text{Resistance}$$

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- How do assumptions about air resistance affect the terminal velocity of a falling object?

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Physical inputs:

- $v = \frac{dx}{dt}$ - velocity is the rate of change of position
- Gravitational force = $\frac{Gm_1m_2}{r^2}$
  - If close to earth, assume $Gm_1/r^2 \approx g$ constant.
- Air resistance: proportional to v or v^2

  (in opposite direction to motion)

Question:

- How do assumptions about air resistance affect the terminal velocity of a falling object?

Question:

- Suppose a lunar lander is falling towards the moon.
- We want the velocity at landing to be 0.
- When should we turn on the upwards thrusters?

Question:

- How fast do we have to launch from the ground to escape the earth's orbit?

$$\frac{dv}{dt} = \text{External Force} + \text{Gravity} + \text{Resistance}$$

Suppose that

- There is no external force
- Gravity is constant
- Resistance is proportional to v

- Can solve the separable equation for $v(t)$
  - Can find y(t)
- Can also analyze using techniques from Ch2.2
- Main qualitative phenomenon: terminal velocity is mg/k.

$$\frac{dv}{dt} = \text{External Force} + \text{Gravity} + \text{Resistance}$$

Suppose that
- There is no external force
- Gravity is constant
- Resistance is proportional to v^2

Careful about the signs!

- Can solve the separable equation for $v(t)$
  - Can find y(t)
- Can also analyze using techniques from Ch2.2
  - Main qualitative phenomenon: terminal velocity is $\sqrt{mg/k}$.

$$\frac{dv}{dt} = \text{External Force} + \text{Gravity} + \text{Resistance}$$

Suppose that

- External force is 0
- Gravity = $\dfrac{GM}{r^2}$
- Resistance 0

Question:

- How fast do we have to launch from the ground to escape the earth's orbit?

$$\frac{dv}{dt} = \text{External Force} + \text{Gravity} + \text{Resistance}$$

Suppose that
- External force is 0 or T
- Gravity = $\frac{GM}{r^2}$
- Resistance 0

Set up differential equations before and after thrust is activated:

Before thrust:

Before thrust:

Question:

- Suppose a lunar lander is falling towards the moon.
- We want the velocity upon landing to be 0.
- When should we turn on the upwards thrusters? Assume that they provide constant thrust.

Assume:
- Initial altitude is y(0)=53km
- Initial velocity is v(0)=1477km/h downwards.
- Thrusters give $T = 4m/s^2$ deceleration
- Mass of moon is $M = 7.35 \cdot 10^{22}$ kg
- Radius is $R = 1740$ km

Consider:

$$\frac{dx}{dt} = x(4 - x) - h$$

How does the qualitative nature of the solutions change when we modify h?

https://www.desmos.com/calculator/u3x5w62sde

One interpretation: How does the number of equilibrium points depend on h?

A bifurcation point for a parameter is a point at which the number of equilibrium solution changes.

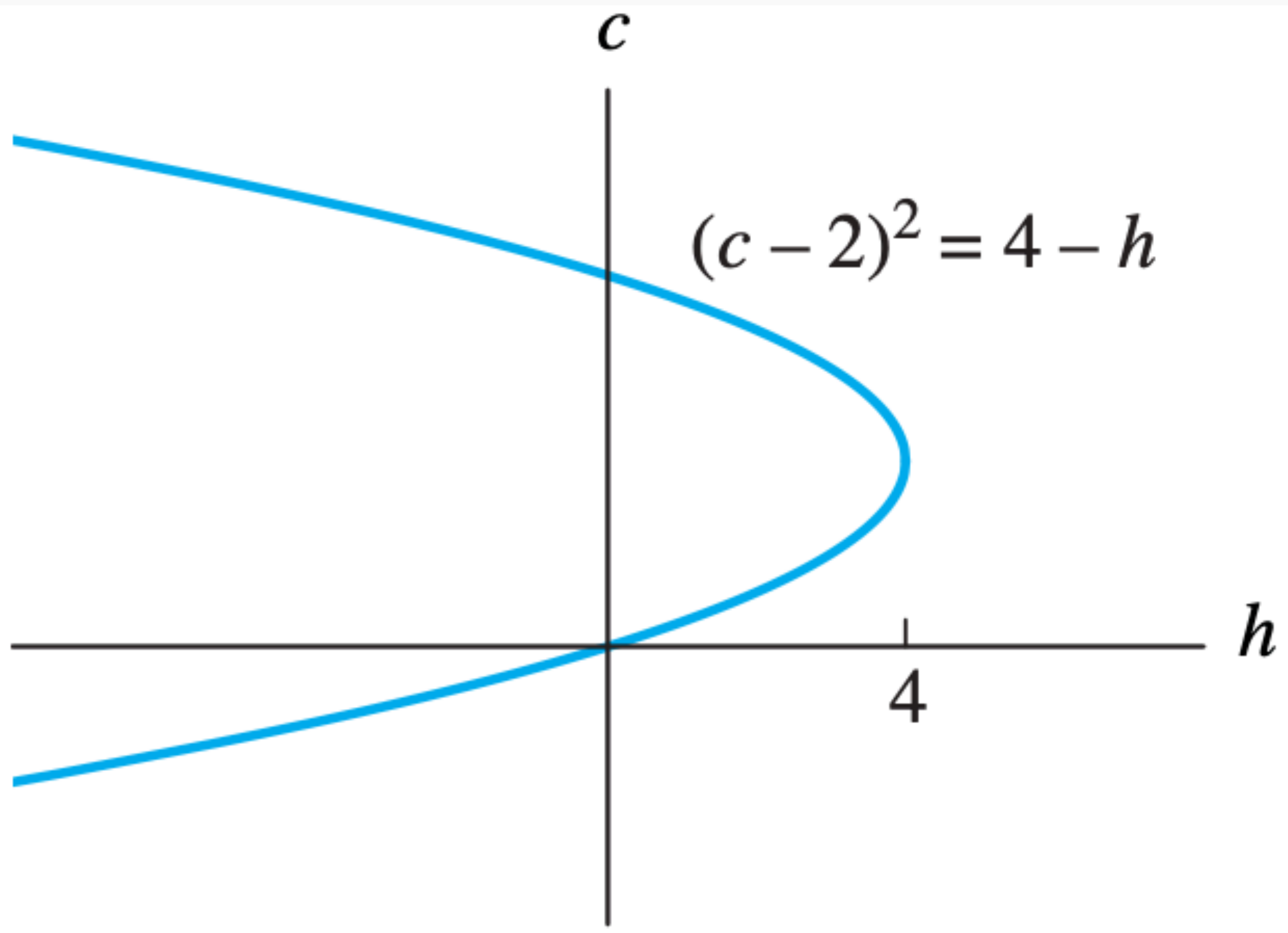

FIGURE 2.2.12. The parabola $(c-2)^2 = 4 - h$ is the bifurcation diagram for the differential equation $x' = x(4-x) - h$.

Bifurcation diagram is a plot of parameter vs. the location of the equilibrium points.

Consider:

$$\frac{dx}{dt} = kx - x^3$$

Draw the bifurcation diagram.