MAT303: Calc IV with applications

Lecture 7 - February 24 2021

Midterm 1:

- Next Wednesday in lecture, proctored over zoom
- Allowed: textbook, lecture notes
- You will also be given a table of useful integrals.
- A random selection of students will be asked to set up a 10 minute meeting with me in the week after the exam to discuss their solutions
  - It is only to verify that you did not cheat
  - It's not meant to be very intense, it's usually pretty easy to determine between
    - Someone who cheated
    - Someone who did not

Last time: Ch 2.1 population models

$$\frac{dP}{dt} = (\beta - \delta)P$$

We saw how changing β, δ affected the behavior of the solution.

Today:

Ch 2.2 Analysis of $\dfrac{dP}{dt} = f(P)$

Example: Let k be a constant. What can we say about the solution to

$$\frac{dx}{dt} = -k(x - A)$$

Solution:

$$x(t) = A + \left(x_0 - A\right) e^{-kt}$$

Conclusion:

- As $t \to \infty$, temperature x approaches A.
- Also, $x = A$ is a solution.

Actually, can find equilibrium solution just by setting $\dfrac{dx}{dt} = 0$:

Example: Let k be a constant. What are the equilibrium solutions?

$$\frac{dx}{dt} = kx(M - x)$$

Question: As $t \to \infty,$ what is the long run behavior of the system?

Question: As $t \to \infty,$ which equilibrium solution do we approach?

Definition: An equilibrium solution $x = c$ to $\frac{dx}{dt} = f(x)$

is **stable** if:

Solutions x(t) starting near c end up staying near c in the long run.

Definition: An equilibrium solution $x = c$ to $\frac{dx}{dt} = f(x)$

is **unstable** if it is not stable.

Example: Let k be a constant. Equilibrium solutions are x=M and x=0.

Are they stable?

$$\frac{dx}{dt} = kx(M - x)$$

Definition: An equilibrium solution $x = c$ to $\frac{dx}{dt} = f(x)$

is **stable** if:

Solutions x(t) starting near c end up staying near c in the long run.

Definition: An equilibrium solution $x = c$ to $\frac{dx}{dt} = f(x)$

is **unstable** if it is not stable.

Example: What are the equilibrium solutions? Are they stable?

$$\frac{dx}{dt} = x^2 - 5x + 4$$

Example: Let k, M, h be constant.

What are the equilibrium solutions?

Are they stable?

$$\frac{dx}{dt} = kx(M - x) - h$$

Consider:

$$\frac{dx}{dt} = x(4 - x) - h$$

How does the qualitative nature of the solutions change when we modify h?

https://www.desmos.com/calculator/u3x5w62sde

One interpretation: How does the number of equilibrium points depend on h?

A bifurcation point for a parameter is a point at which the number of equilibrium solution changes.

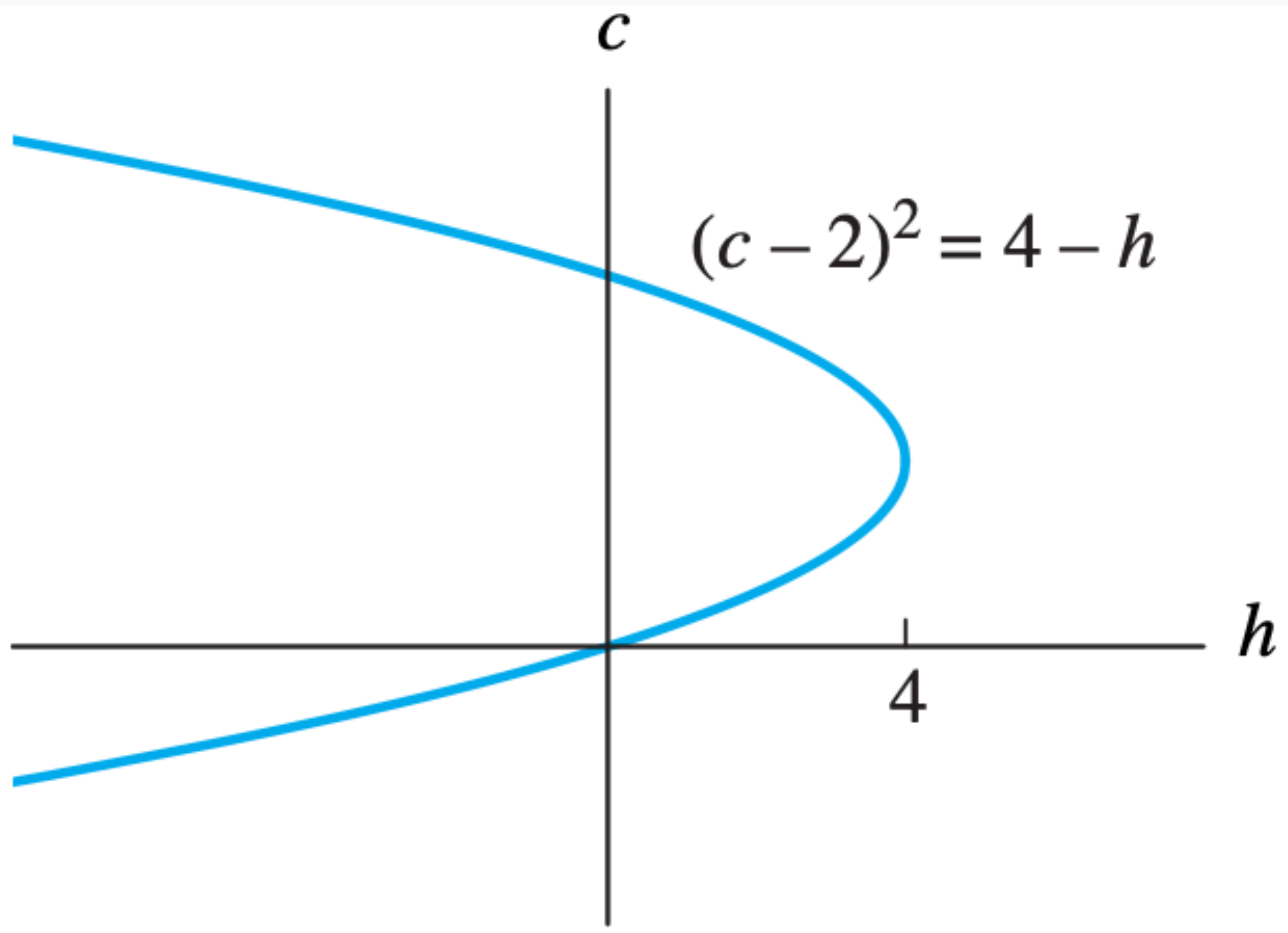

FIGURE 2.2.12. The parabola $(c - 2)^2 = 4 - h$ is the bifurcation diagram for the differential equation $x' = x(4 - x) - h$.

Bifurcation diagram is a plot of parameter vs. the location of the equilibrium points.