MAT303: Calc IV with applications

Lecture 2 - February 08 2021

Last time:

- Different ways of interpreting functions and DEs
- Slope field

Today:

- Separable equations
- Solving problems using separable equations. (Applications)

What is the point of solving differential equations:

- Answer questions like
  - What is $y(4)$?
  - What is $y(t)$?
- Get an idea of what the behavior of the model
- See what happens when initial condition changes
- See what happens in the long run
- See what happens when model changes

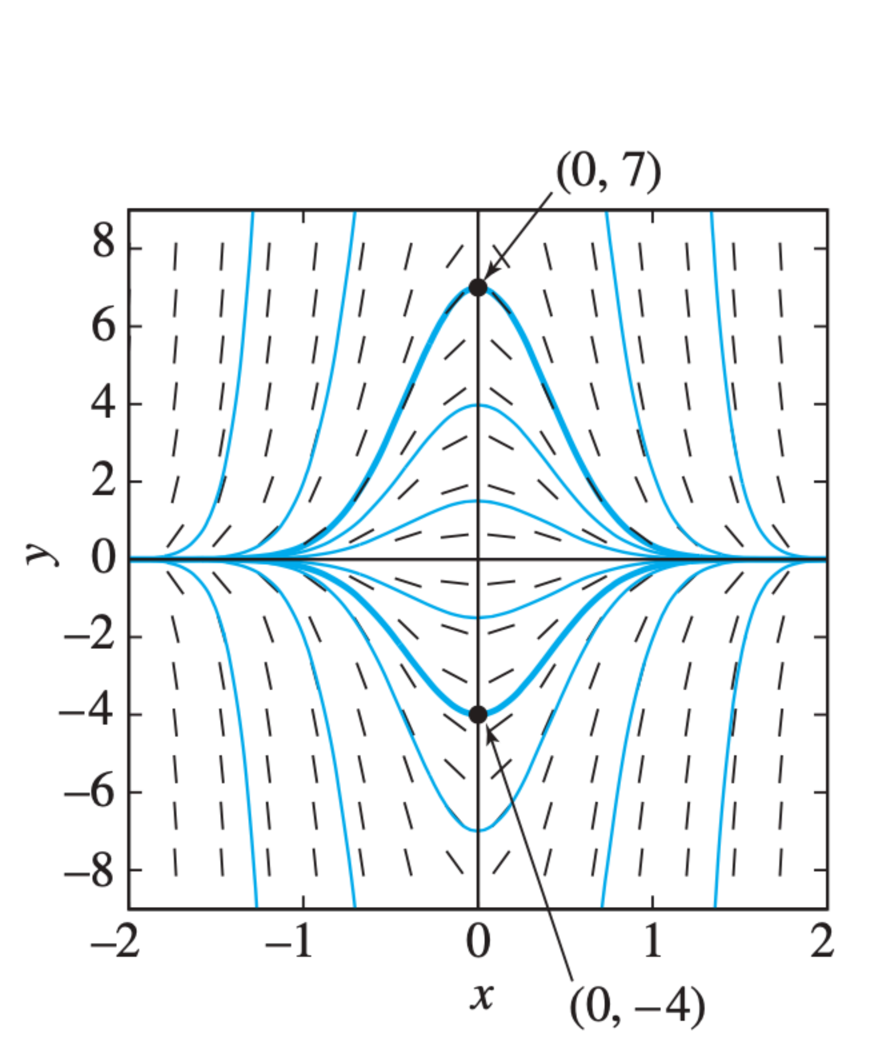
First order equation where RHS does not depend on y:

$$\frac{dy}{dx} = -6x$$

First order equation where RHS does depend on y:

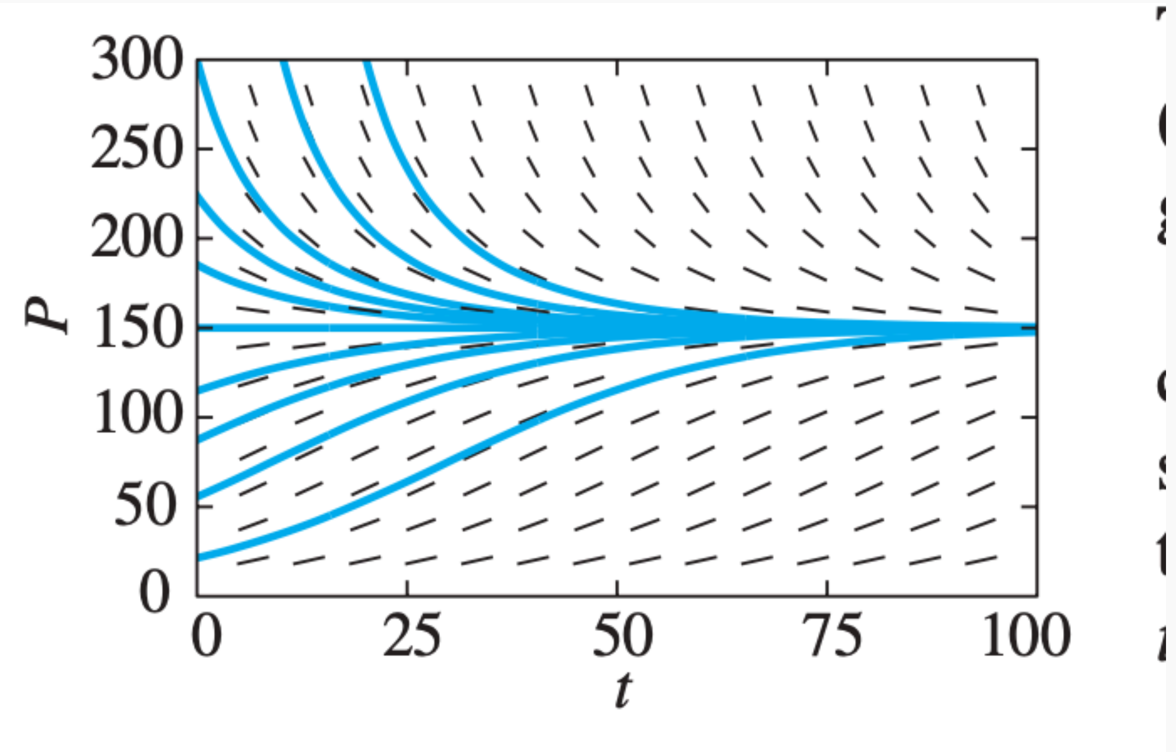
$$\frac{dy}{dx} = -6xy$$

New technique: separation of variables



Example 2: logistic population growth
(From lecture 2), HW)

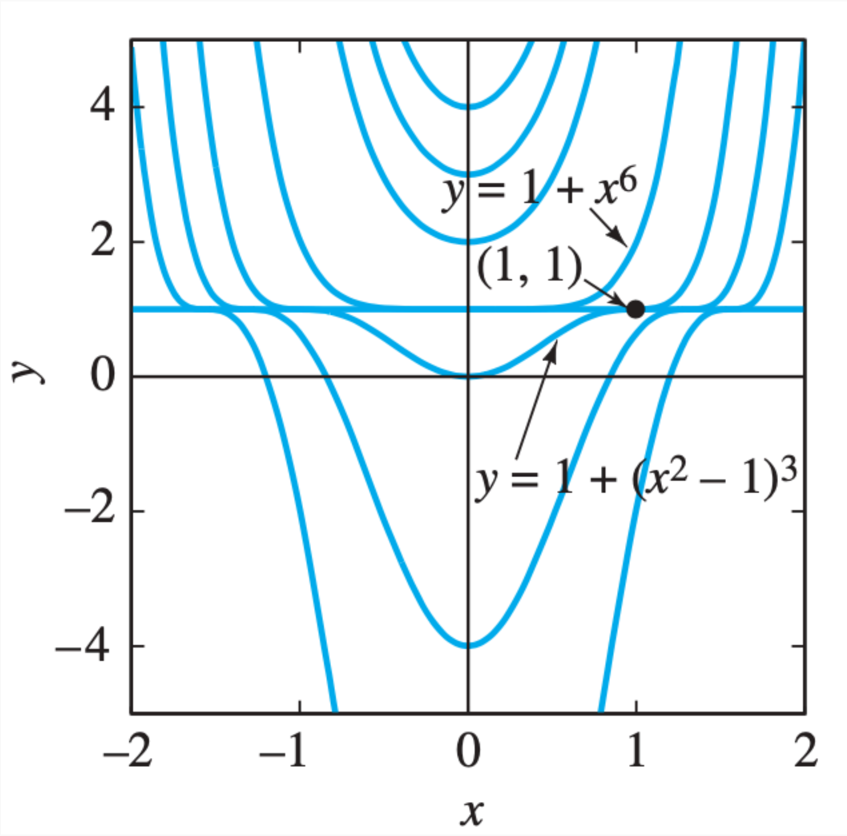
$$\frac{dP}{dt} = kP - lP^2,$$
$$k = 0.0225, \quad l = 0.0003$$
$$P(0) = 25$$



Example 3: from textbook

Solve for y:

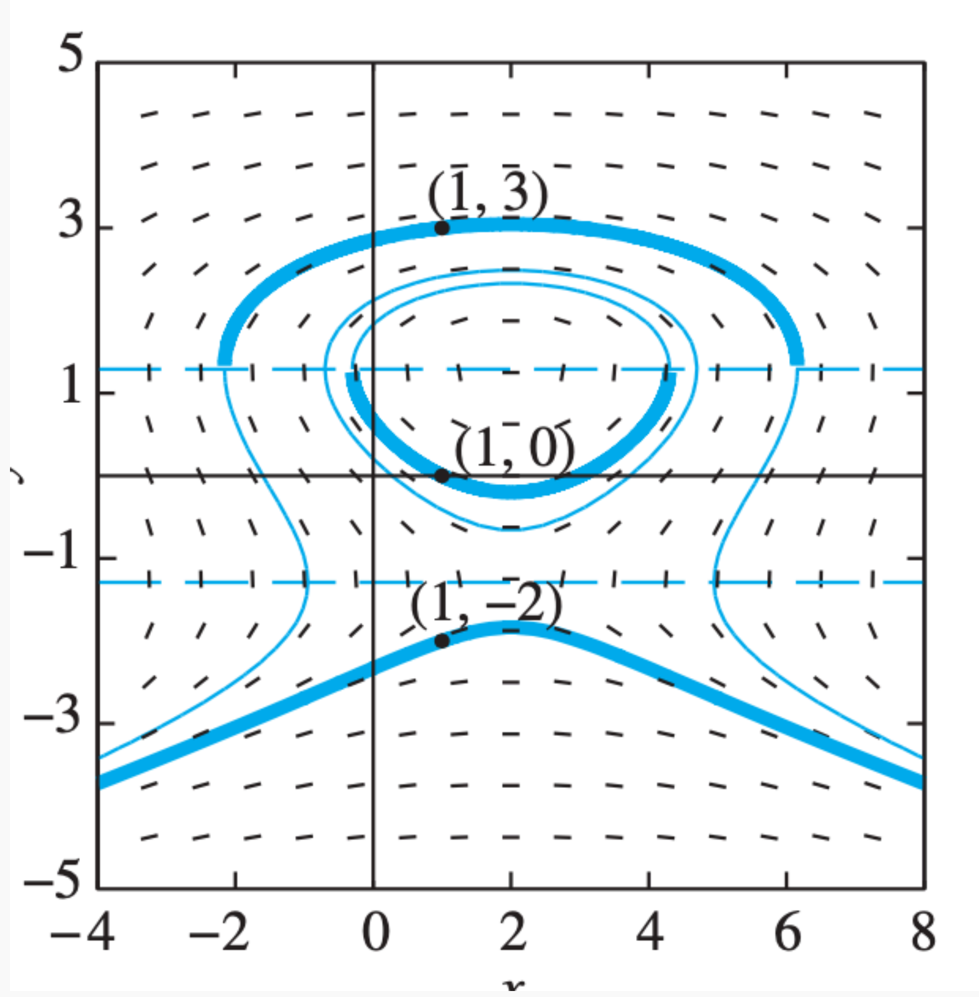
$$y' = 6x(y-1)^{2/3}$$



$x^2 + y^2 = C$

Example 3 (from textbook)

$$y' = \frac{4-2x}{3y^2-5}$$



When does separation of variables work?

a) $\frac{dy}{dx} = ye^x$

b) $\frac{dy}{dx} = \cos(y)\sin(x)$

c) $y' = \cos(e^x + y)$

d) $y' = y$

e) $y' = \frac{4 - 2x}{3y^2 - 5}$

Example applications:

Radioactive decay:
Let N(t) be the number of atoms of a certain radioactive isotope at time t.
Then:
$$\frac{dN}{dt} = -kN$$

This is used for radiocarbon dating:

- radioactive carbon stays constant for living organism
- When organism dies, this decays according to the differential equation above
- So we if we measure the radioactive carbon content of an object, we can figure out how long ago it died.

Example:

- Charcoal at Stonehenge contains 63% as much radioactive carbon compared to present-day charcoal of the same mass.
- How old is the sample?
- Assume that radioactive carbon decays according to the differential equation

$$\frac{dN}{dt} = -kN, \quad k = 0.0001216$$

Solution:

- Suppose you have water in a tank, of depth y
- Tank has a hole at the bottom.
- Then the velocity of the water flow at the bottom is

$$v = \sqrt{2gy}$$

Question: Suppose you have a hemispherical bowl has radius 4ft and it is full of water. Suppose you open up a hole of diameter 1in at time $t = 0$.

1. Write down a differential equation for the water depth y.
2. What is the depth of water at time t?
3. How long does it take the tank to empty?

Key idea: we need to find a differential equation for the volume V.

Rate of change in volume = cross sectional area * velocity

See textbook and hw for other applications such as

- **Drug elimination**
- **Population Growth**
- **Newton’s law of cooling**
- **Compound interest**
- **Water flow in a tank**
- **Logistic population growth**
- **Radioactive decay**