MAT303: Calc IV with applications

Lecture 2 - February 08 2021

Recall:

- What is a differential equation
- Why we should study differential equations
- Ch1.1: Differential equations and mathematical models
- Ch1.2: Integrals as solutions to differential equations

What/why:

- Many processes in the world can be described by their rate of change
- Rate of change <-> derivative
- Equations involving derivatives are *differential equations*.
- Differential equations allow us to study mathematical models of physical processes.

Today:

- Different ways of interpreting functions and DEs
- Slope field

Advantages of multiple interpretations

- More opportunities to see when DEs are useful
- Easy to reason about general properties of DEs
- Easy to reason about specific DEs

We will see:

- Why most DE has infinitely many solutions
- Why adding an initial condition makes it unique

First order equation where RHS does not depend on y:

First order equation with initial condition:

First order differential equation:

$$\frac{dy}{dx} = f(x, y)$$

Draw the slope field:

$$\frac{dy}{dx} = x - y$$

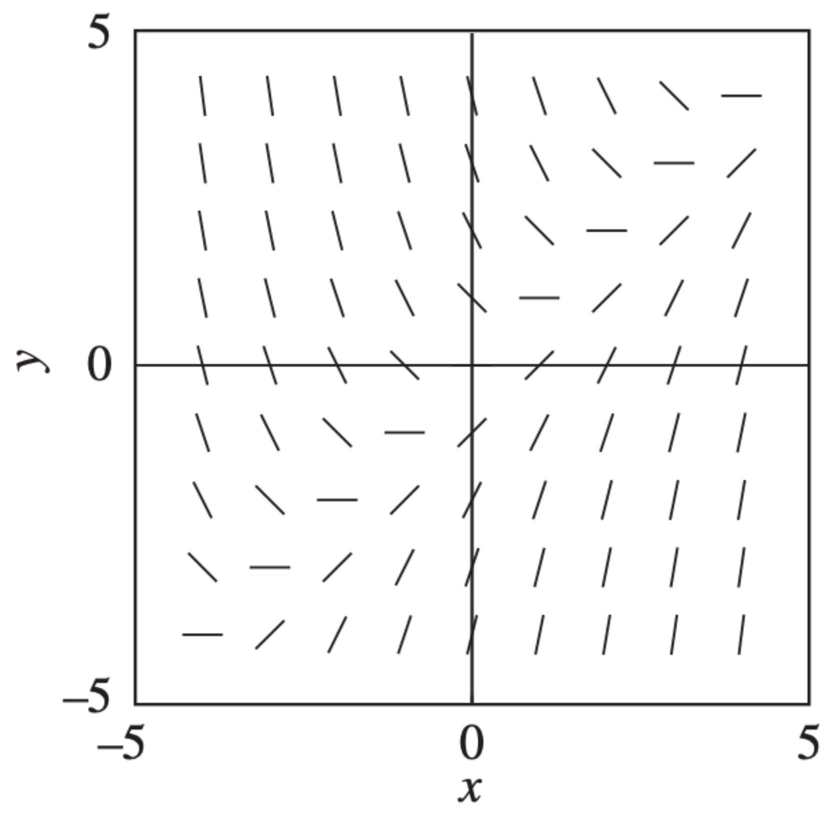


FIGURE 1.3.4. Slope field for $y' = x - y$ corresponding to the table of slopes in Fig. 1.3.3.

y(-4)=4

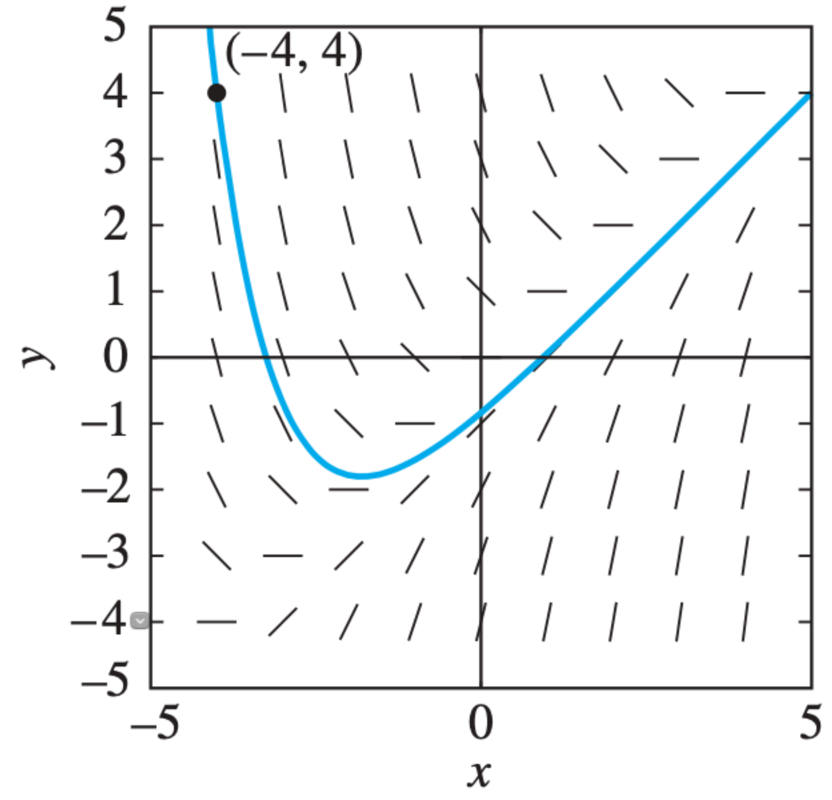
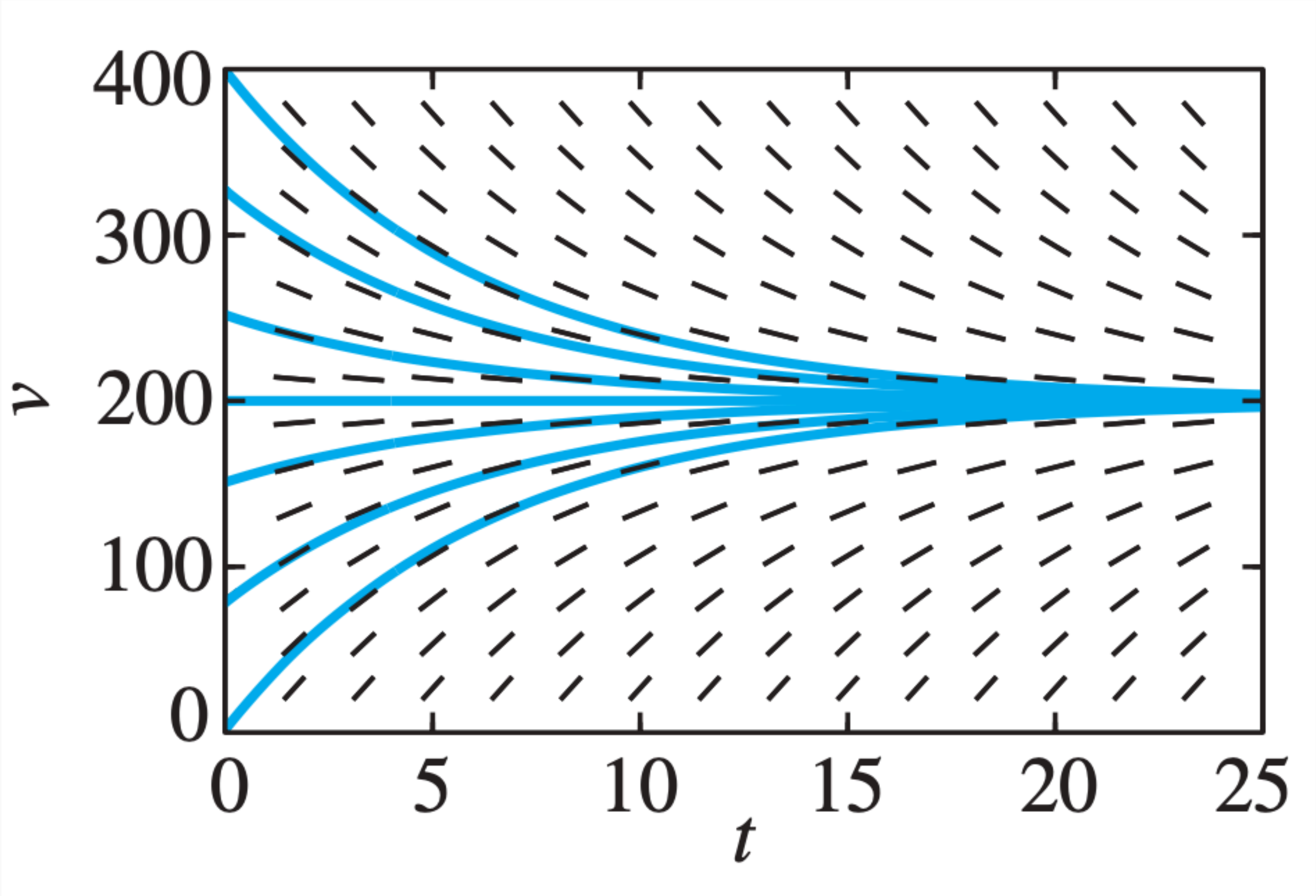


FIGURE 1.3.5. The solution curve through $(-4, 4)$.

Air resistance is proportional to velocity:

$$\frac{dv}{dt} = g - kv$$

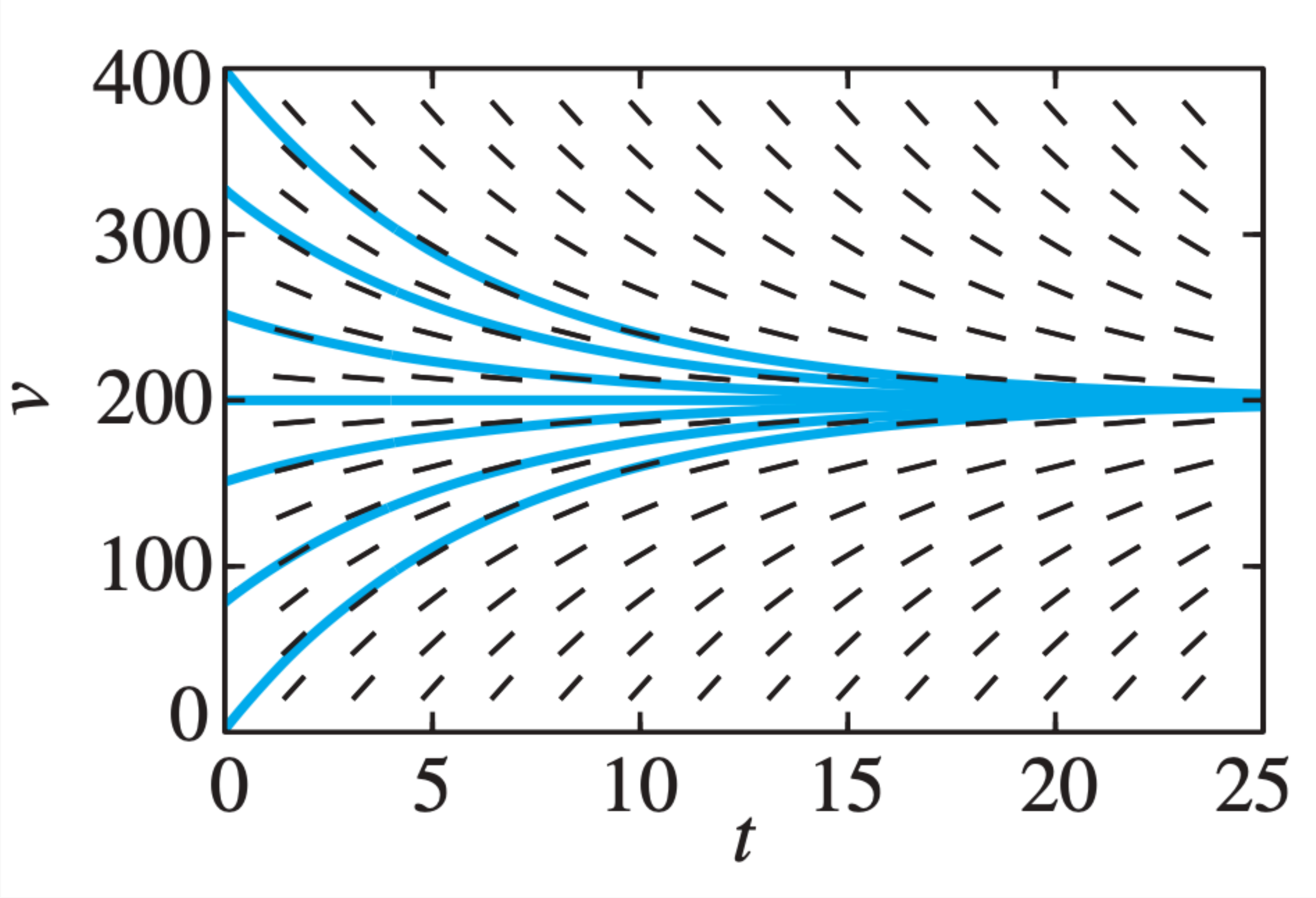
$$\frac{dv}{dt} = 32 - 0.16v$$



Logistic model for population growth:

$$\frac{dP}{dt} = kP(M - P)$$

$$\frac{dP}{dt} = 0.0004P(150 - P)$$



Intuitively:

- Differential equations usually have infinitely many solutions
- Adding an initial condition usually narrows it down to a unique solution

The technical statement:

THEOREM 1 Existence and Uniqueness of Solutions

Suppose that both the function $f(x, y)$ and its partial derivative $D_y f(x, y)$ are continuous on some rectangle R in the xy-plane that contains the point (a, b) in its interior. Then, for some open interval I containing the point a, the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(a) = b \tag{9}$$

has one and only one solution that is defined on the interval I. (As illustrated in Fig. 1.3.11, the solution interval I may not be as "wide" as the original rectangle R of continuity; see Remark 3 below.)

Example:

$$x\frac{dy}{dx} = 2y$$