MAT303: Calc IV with applications

Lecture 1 - February 03 2021

MAT303: Differential Equations

Today:

- What is a differential equation
- Why we should study differential equations
- Ch1.1: Differential equations and mathematical models
- Ch1.2: Integrals as solutions to differential equations

What/why:

- Many processes in the world can be described by their rate of change
- Rate of change <-> derivative
- Equations involving derivatives are *differential equations*.
- Differential equations allow us to study mathematical models of physical processes.

Example:

- Population grows at a rate proportional to the current population
- Derivative of population is proportional to the current population
- $\frac{dP}{dt} = kP$ <- Looking for function whose derivative is k times itself
- Solution is ?

The problem is that it is easy to describe,
but hard to answer questions like ‘what is the population at time 10’?

Analogy:

- Some “real world” problem
- .
- $x^2 - 3x + 2 = 0$
- Solutions are $x = 2$ and $x = 1$

Similarities:

- There can be multiple solutions
- There can be no solutions
- It is easier to verify a solution that it is to find it.

Differences:

- For an algebraic equation, we are looking for a **number** to solve the equation
- For a differential equation, we are looking for a **function** to solve the equation

Example:

- Population grows at a rate proportional to the current population
- Derivative of population is proportional to the current population
- $\dfrac{dP}{dt} = kP$
- Solution is $P(t) = Ce^{kt}$, where C is any constant.

- How to interpret a differential equation
  - How to connect the differential equation to what you are modeling
  - How to go from a model to a differential equation
- Some techniques for solving the most basic and common differential equations
  - See how the setup is reflected in the solution

Ideally, after this class,

- You will be able to recognize the most basic and common DEs
- You will recognize when it is suitable to model something as a DE
- You will not be surprised when DEs come up in your work

Main resource: course website

http://www.math.stonybrook.edu/~bplin/teaching/spring2021/mat303/index.html

Syllabus, Lectures notes, schedule, hw, etc.
It is your responsibility to check this regularly.

Two services to sign up for. Links are on course website.

- Piazza: discussion board and announcements
- Gradescope: you must submit your homework here

Assessment (see syllabus)

- Homework (30%): due most Wednesdays. First one due Feb 10.
- Midterms (40%): March 3 and April 7 in class.
- Final (30%): May 18 5:30pm-8pm.

Newton's law of cooling:
The rate of change of the temperature T of a body is proportional to the difference between between T and the temperature A of the surrounding medium.

Differential equation:

$$\frac{dT}{dt} = k(T - A)$$

Notice how this is much more compact and clearer than the description in words.

Solution:

$$T = A + Ce^{-kt}$$

A,C,k are constants.

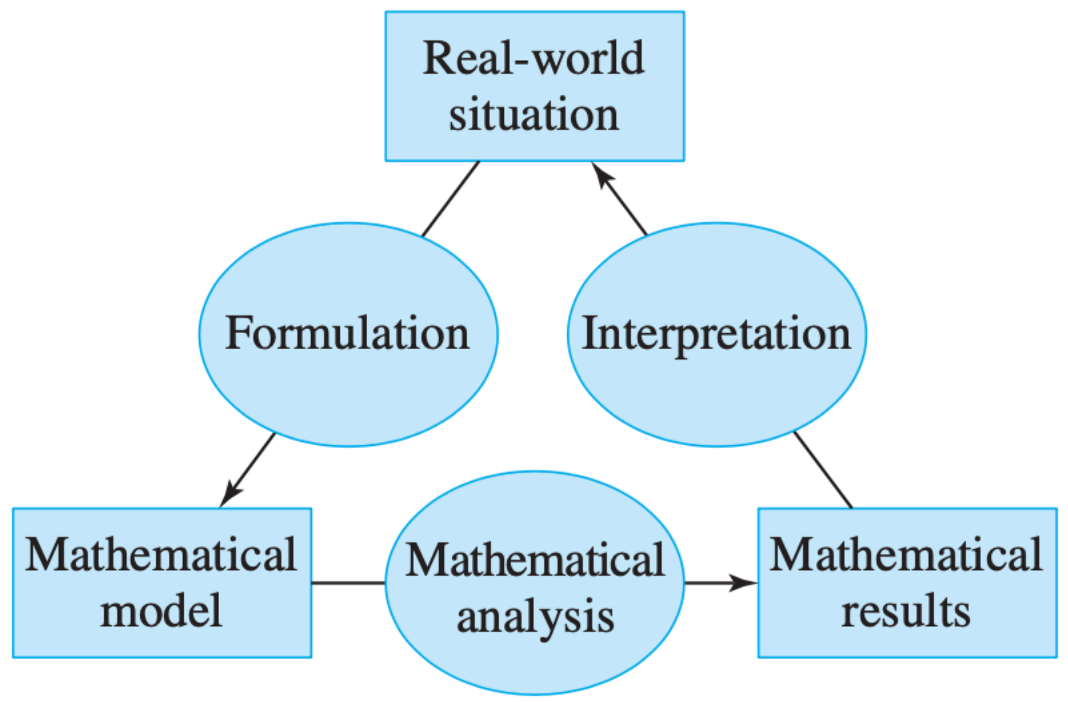

FIGURE 1.1.4. The process of mathematical modeling.

In the previous example,

$$\frac{dT}{dt} = k(T - A)$$

$$T = A + Ce^{-kt}$$

There were infinitely many solutions, depending on the constant C.
How do we choose the correct solution to correspond to our physical model?

Answer: choose the correct initial conditions.

T(0)=A+C, so C = T(0)-A.

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Another example: verify that $y(x) = 2x^{1/2} - x^{1/2} \ln x$

Is solution to the differential equation with initial value

$$4x^2 y'' + y = 0$$
$$y(0) = 0$$

Simplest type of differential equation:

$$\frac{dy}{dx} = f(x)$$

Example:

$$\frac{dy}{dx} = 2x + 3, \quad y(1) = 2$$

Simplest type of differential equation:

$$\frac{dy}{dx} = f(x)$$