MAT200: Logic Language and Proof

Lecture 1 - February 03 2021

- Basic building blocks of math
  - Propositions, Statements, Theorems, Definitions
  - Sets, functions, numbers
- How to write *proofs*.
  - Techniques: Contradiction, Induction, …
- Some mathematical facts we will prove:
  - (-1)*(-1)=1
  - A number is divisible by 3 if and only if the sum of its digits is divisible by 3
  - If your sock drawer has 12 black and 12 blue socks, how many socks do you need to pick out to be guaranteed a matching pair? Only 3.
  - If you pick any 10 numbers from {1,….,100}, you can always find two groups of numbers whose sum is equal.
  - $1 + 4 + 9 + 16 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
  - Some real numbers cannot be written as fractions.
  - Some real numbers cannot be written as roots of polynomials.

3721

$3+7+2+1 = 13$

so 3721 not divis by 3.

4^2

$1+4+9+16 = 30$

$\frac{n(n+1)(2n+1)}{6} = \frac{4(4+1)(2\cdot 4+1)}{6} = \frac{4\cdot 5\cdot 9}{6} = \frac{180}{6} = 30.$

Essentially, these are the skills we want you to have after this class:

- How to understand mathematical statements, or
  - How to turn vague ideas into precise statements
- How to write proofs of said statements
  - Coming up with the idea
  - Presenting it so that others can understand
- How to read proofs of said statements
  - Finding mistakes/omissions
  - Understanding how the author came up with the solution

- Reading and writing mathematical proofs teaches you how to think.
- There are no ‘real world’ distractions.

Main resource: course website

http://www.math.stonybrook.edu/~bplin/teaching/spring2021/mat200/index.html

Syllabus, Lectures notes, schedule, hw, etc.
It is your responsibility to check this regularly.

Two services to sign up for. Links are on course website.

- Piazza: discussion board and announcements
- Gradescope: you must submit your homework here

Assessment (see syllabus)

- Homework (30%): due most Wednesdays. First one due Feb 10.
- Midterms (40%): March 8 and April 7 in class.
- Final (30%): May 18 2:15pm-5pm.

A *proposition* is a sentence which is either true or false.

Examples:

* $2+2=4$ T ← NOT $2+2\neq 4$ F
* $2+2=5$ F ← $2+2\neq 5$ T
* The area of a circle of radius r is πr^2. ← Area of circle is not πr^2.
* For every real number x, $x^2\geq 0$. ← There is some real number x, $x^2<0$
* Every even integer except 2 is the sum of 2 prime numbers. E.g. $12=5+7$

Not proposition:

* $3-4$
* Let $x=5$

A *predicate* is a proposition that depends on some variables.

Examples:

* $n>3$ T/F depends on what n is.
* $n^2-2n>0$
* $m<n$
* $e^x<3$.
* n is even

A *statement* is a proposition or a predicate.

(Notice how we are already starting to define things in terms of other things that we have defined)

We all know that we can apply operations to numbers, to get new numbers.

$$3 + 7 = 10$$

$$3 \times 7 = 21$$

$$x + y \doteq x \text{ plus } y.$$

$$-3 = \text{negative } 3.$$

3 plus 7 = 10

3 times 7 = 21

There are also operations that we can apply to functions, to get new functions.

$$f(x) = x^2$$

$$\frac{df}{dx} = 2x.$$

$\frac{d}{dx}$ is an operator.

There are also operations that we can apply to $\overbrace{\text{propositions and predicates}}^{\text{statements}}$, to get new $\underbrace{\text{propositions and predicates}}_{\text{statements}}$.

Given two statements P,Q,
‘P ^ Q’ is another statement.
It is sometimes written ‘P and Q’

$$P = F$$

$$Q = F$$

$$P \wedge Q = F.$$

~~Quiz~~: if P = “3 is even” and Q=“π is an integer”

P∧Q = “3 is even and π is integer”

There are also operations that we can apply to propositions and predicates, to get new propositions and predicates.

Given two statements P,Q,
'P ^ Q' is another statement. It means P and Q.
It is sometimes written 'P and Q'

E.g. P = "student did homework" and Q="student went to class"
E.g. P = "4 is even", Q = "4 is prime"

$4 = 2 \times 2$.

$P = T$

$Q = F$

$P \wedge Q =$ "4 is even and 4 is prime"

a) T
b) F

Given two statements P,Q,
'P v Q' is another statement.
It is sometimes written 'P or Q'

There are also operations that we can apply to propositions and predicates, to get new propositions and predicates.

$\wedge$ = and

$\vee$ = or

Given two statements P,Q,
'P v Q' is another statement. It means P or Q.
It is sometimes written 'P v Q'

E.g. P = "student did homework" and Q="student went to class"
E.g. P = "4 is even",   Q = "4 is prime"

R = "6 is even"

P v Q = "4 is even or 4 is prime" = T

P v R = "4 is even or 6 is even" = T

T = "4 is odd"

T v Q = "4 is odd or 4 is prime" = F

Notice: the truth value of P v Q only depends on the truth value of P and the truth value of Q.

You don't need to know what P and Q are to determine whether P v Q is true.

Can represent this in a truth table.

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F

Can use *truth tables* as a **definition** for the logical connective.

There are also operations that we can apply to propositions and predicates, to get new propositions and predicates.

Apply "not" to a statement is sometimes called negating the statement.

Given a statement P,
‘not P’ means not P.

P = 4 is even

(not P) = 4 is not even.

P = student completed all hw

(not P) = (student didn't complete all hw).

Truth table:

P	not P
T	F
F	T

and

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Notice how, by drawing the truth table, we define what the operator does instead of having to resort to English.

Quiz: if P = $\underbrace{\text{“3 is even”}}_{F}$ and Q=$\underbrace{\text{“}\pi\text{ is an integer”}}_{F}$

F F
P v Q = "3 is even or π is integer" = F

F F
P ^ Q = "3 is even and π is integer" = F

not $(\underbrace{P \wedge Q}_{F})$ = T

(not P) v (not Q) = T
T T

(not P) ^ (not Q) = T
T T

not (P v Q) = T
F

Boolean algebra.

Quiz: if P = “student did homework” and Q=“student went to class”

P v Q =

P ^ Q =

not (P ^ Q) =

(not P) v (not Q) =

(not P) ^ (not Q) =

not (P v Q)

Just like there are identities between operators on numbers, there are identities between logical operations on statements.

$$a(b+c) = ab + ac$$

De Morgans laws:

$$\text{not}\,(P \vee Q) = (\text{not}\,P) \wedge (\text{not}\,Q)$$

$$\text{not}\,(P \wedge Q) = (\text{not}\,P) \vee (\text{not}\,Q)$$

How to prove? Use truth table.

$P =$ "student did hw"

$Q =$ "student came to class"

$\text{not}\,(P \vee Q) =$ not (student did hw or student came to class)

$(\text{not}\,P) \wedge (\text{not}\,Q) =$ student didn't come to class and didn't do HW.

Next class: we will prove these identities

P or (Q and R)

3 − 4 × 5