Problem 1 (10 points)

(a) Prove that: if a is even and b is even, then $a + b$ is even.

(b) Prove that: if a is odd and b is odd, then $a + b$ is even.

Problem 2 (10 points)

(a) Consider the following statement, P.

If n^2 is even, then n is even.

Find the (first) mistake in the following proof of P and explain why it is a mistake.

> **Proof:** Suppose n is even. Then $n = 2q$ for some integer q. Therefore $n^2 = 4q^2$. Therefore $n^2 = 2k$ where k is integer. Therefore n^2 is even.

(b) Is the statement, P, true or false? Why?

Problem 3 (10 points)

Consider the following statement, P.

$$\text{If } a \text{ is real, } a \cdot 0 = 0.$$

Find the (first) mistake in the following proof of P and explain why it is a mistake.

Proof: We have $1 \cdot 0 = 0$ by the unity law (for any x, $1 \cdot x = x$). Therefore, for any a, we have

$$\begin{aligned} a \cdot 0 &= a \cdot (1 \cdot 0) \\ &= (a \cdot 1) \cdot 0 \\ &= a \cdot 0 \\ &= 0 \end{aligned}$$

which is what we wanted to show.

Problem 4 (6 points)

The following is a proof that $a \cdot 0 = 0$. The author has assumed that the reader can justify each step. Your task is to justify each step. (Most of the steps will involve citing the correct axiom(s) from the real number axioms).

Proof: Let a be any real number.

1. We have $0 + 0 = 0$.
2. Therefore $a \cdot (0 + 0) = a \cdot 0$.
3. Therefore $a \cdot 0 + a \cdot 0 = a \cdot 0$.
4. Therefore $a \cdot 0 + a \cdot 0 + (-a \cdot 0) = a \cdot 0 + (-a \cdot 0)$.
5. Therefore $a \cdot 0 + 0 = 0$.
6. Therefore $a \cdot 0 = 0$, which is what we wanted to prove.

Problem 5 (10 points)

For the following problems, use [1, Axioms 2.3.1]. When you use an axiom, specify which axiom you are using. You can also use the fact that $a \cdot 0$ for any a, which is proved elsewhere in this homework.

(a) Prove that if a is a real number, $(-a) = (-1) \cdot a$. Hint: To prove this, you want to show that the object $(-1) \cdot a$ satisfies the same defining property as $(-a)$.

(b) Prove the following statement: $x = 0 \implies x^2 = 0$.

(c) What is the converse of the preceding statement? Prove this converse. Hint: There are two ways to proceed: either use proof by contradiction (read ahead) or use the order axioms 3.1.2.

Problem 6 (10 points)

For the following problems, you must use [1, Axioms 2.3.1]. Clearly justify each of your steps by referring back to the axioms. You can also use anything that we have already proved elsewhere in this homework.

(a) Prove that, for all real numbers a, we have $-(-a) = a$.

(b) Prove that $(-1) \cdot (-1) = 1$.

References

[1] P. J. Eccles, *An Introduction to Mathematical Reasoning: Numbers, Sets and Functions.* Cambridge University Press, 1997.