# MAT200: Logic Language and Proof

Lecture 9 - March 3 2021

**Midterm 1:**

- Next Wednesday in lecture, proctored over zoom
- Covers everything up to and including this lecture.
  - See HW, and “practice problems” on website for practice problems.
- Allowed: textbook, lecture slides, your own notes based on the lecture notes
- A random selection of students will be asked to set up a 10 minute meeting with me in the week after the exam to discuss their solutions
  - It is only to verify that you did not cheat
  - It’s not meant to be very intense, it’s usually pretty easy to determine between
    - Someone who cheated
    - Someone who did not

Recently: sets

Today: Quantifiers

P(N)=$\forall x \in \mathbb{Z} : 1 < x < N \implies \neg(\exists d \in \mathbb{Z}, \quad dx = N)$.

$\forall N \in \mathbb{Z}, \exists M \in \mathbb{Z} : M > N$ and $P(M)$

$\forall$ means "For all"

$\exists$ means "There exists"

We've seen some examples of ¬ applied to statements.

It can be a bit tricky, so here are some tips to check if you negated it correctly:

- Only one of the sentences $P, \neg P$ can be true.
- One of $P, \neg P$ should be true (even if you don't know which one).
- When $P$ is true, $\neg P$ should be false, and vice versa.

Example: $P =$ Everyone has a valid ticket
Which one is correct?

$\neg P =$ No-on has a valid ticket
$\neg P =$ Someone has a valid ticket
$\neg P =$ Everyone does not have a valid ticket
$\neg P =$ Someone  does not have a valid ticket

With quantifiers:

Let Q(x) be the statement "Person x has a valid train ticket"

In general:

There is a mechanical way to take negations of nested quantified statements.

- $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- $\varnothing = \{\}$
- $\mathscr{P}(A) = \{X : X \subset A\}$
- $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$
- $A - B = \{x : x \in A \text{ and } x \notin B\}$
- $A^c = U - A$

Theorem:

$$\forall x \in \mathbb{R} : x^2 \geq 0$$

Theorem:

$$\exists x \in \mathbb{R} : x^2 = 9$$

True or false?

$$\exists x \in \mathbb{R} : x^2 = 1$$

Theorem:

$$\forall n \in \mathbb{Z} : ((\exists q \in \mathbb{Z} : n = 2q) \implies (\exists q \in \mathbb{Z} : n^2 = 2q))$$

1. P(N)= $\forall x \in \mathbb{Z} : 1 < x < N \implies \neg(\exists d \in \mathbb{Z}, \quad dx = N)$.

   $\forall N \in \mathbb{Z}, \exists M \in \mathbb{Z} : M > N$ and $P(M)$

**Principle of induction:**

Let P(n) be a predicate. If the following statements are true:

- $P(1)$
- $P(n) \implies P(n+1)$

Then the following statement is also true:

- For all $n \geq 1$, we have that $P(n)$ is true.

2. Let A be a subset of positive integers. If

   - $1 \in A$
   - $\forall k \in \mathbb{Z}^+(k \in A \implies k+1 \in A)$

   Then A = $\mathbb{Z}^+$.

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