Lecture 38:

Review. What questions do you have?

Review sessions on Monday May 11

* 10am - 11-30 am
* 1pm - 2:30 pm
* 3pm - 4:30pm

Chain rule.

374 $f(x) = (4-x^2)^3$

Find $f'(x)$.

$$f'(x) = 3(4-x^2)^2 \cdot (-2x)$$

$$= -3(4-x^2)^2(2x)$$

$f(x) = z^3$

because

$$\frac{d}{dz} z^3 = 3z^2$$

(power rule)

When to use log differentiation.

Reason:

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln(a^b) = b\ln(a).$$

Example:-

$$f(x) = \ln\left(\sqrt{5x-7}\right)$$

$$f'(x) = \frac{1}{\sqrt{5x-7}} \frac{1}{2}(5x-7)^{-1/2} \cdot 5.$$

Using log laws:

$$f(x) = \ln\left(\sqrt{5x-7}\right) = \ln\left((5x-7)^{1/2}\right)$$

$$= \frac{1}{2}\ln(5x-7)$$

Now differentiate:

$$f'(x) = \frac{1}{2}\frac{1}{5x-7} \cdot 5.$$

See midterm Q2f)

E.g.

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

Differentiate

$$f(x) = \ln\left(\frac{(5x+1)^2 (\sin x)^{1/2}}{(2x^2+3)(\cos x)} \right)$$

$$= 2\ln(5x+1) + \frac{1}{2}\ln(\sin x) - \ln(2x^2+3) - \ln(\cos x)$$

$$f'(x) = 2\frac{5}{5x+1} + \frac{1}{2}\frac{\cos x}{\sin x} - \frac{4x}{2x^2+3} - \frac{-\sin x}{\cos x}.$$

Actually, you don't even need to a $\ln$, you can introduce it!

Example Logarithmic differentiation.

differentiate.

$$y = \frac{(5x+1)^2 (\sin x)^{1/2}}{(2x^2+3)(\cos x)}$$

Take ln of both sides:

$$\ln y = \ln\left(\frac{(5x+1)^2 (\sin x)^{1/2}}{(2x^2+3)(\cos x)}\right)$$

Use implicit differentiation:

$$y\frac{dy}{dx} = 2\frac{5}{5x+1} + \frac{1}{2}\frac{\cos x}{\sin x} - \frac{4x}{2x^2+3} - \frac{-\sin x}{\cos x}$$

$$\frac{dy}{dx} = 2\frac{5}{5x+1} + \frac{1}{2}\frac{\cos x}{\sin x} - \frac{4x}{2x^2+3} - \frac{-\sin x}{\cos x}$$

y.

Ch 4.5. # 236

Find

a) increasing / decreasing intervals

b) local max / min

c) concave up / down.

d) Sketch graph.

236 $f(x) = \frac{\sin(x)}{x}$

$$f'(x) = \frac{\cos(x)x - \sin(x)}{x^2}$$

$$f''(x) = \frac{\left(\cos(x) - x\sin(x) - \cos x\right)x^2 - \left(\cos(x)x - \sin(x)\right)2x}{x^4}$$

$$= \frac{\cos(x)x^2 - x^3\sin(x) - x^2\cos x - 2x^2\cos(x) - 2x\sin(x)}{x^4}$$

Analysis of f':

$$f' = 0 \iff \cos(x)\,x - \sin(x) = 0.$$

$$\iff \tan(x) = x.$$

$$\iff x \approx 4.5$$

$$\approx x = 0$$

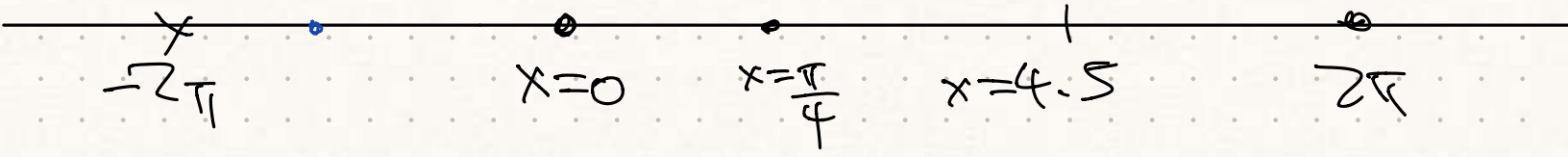
Test points in between $f' = 0$.

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\frac{\pi}{4} - \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\pi}{4} - 1\right) < 0$$

$$f'\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(-\frac{\pi}{4}\right) - \frac{1}{\sqrt{2}} < 0$$

$f' < 0$, $f' = 0$ $\quad f' < 0$ $\quad f' = 0$ $\quad f' > 0$.



Once you have this, it becomes PS on MT2.