Lecture 35

* Final Exam

  Wednesday May 13   8am – 10:45am

  * Same format as midterm.
  * Cumulative.

* Review sheet next week.

* Review session next

  Thu 4pm – 5:20pm

  Fr: 10am – 10:53am

  (in lectures)

  Will be recorded.

Last time: We talked about

$\lim_{x \to \infty} f(x)$ because its neccessary

to evaluate such limits for

unbounded optimization problems.

Quiz:

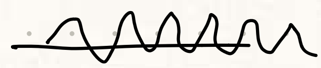
$$\lim_{x \to \infty} \frac{3x^2}{2x+1} =$$

a) 0
b) 1
c) 2
d) 3/2
e) ∞

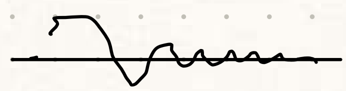
$$= \lim_{x \to \infty} \frac{3x^2}{2x} = \lim_{x \to \infty} \frac{3x}{2} = \infty.$$

Examples from last time:

$$\lim_{x \to \infty} \sin(x) = \text{DNE}$$



$$\lim_{x \to \infty} \frac{\sin(x)}{x^2} = 0$$



$$\lim_{x \to \infty} \frac{3x^2}{2x^2+1} = 3/2.$$

Definition (End behaviour)

A <u>limit at infinity</u> of $f(x)$

is $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$

and it falls into 1 of 3 categories:

① Infinite limit, $\lim_{x\to\infty} f(x) = \infty$

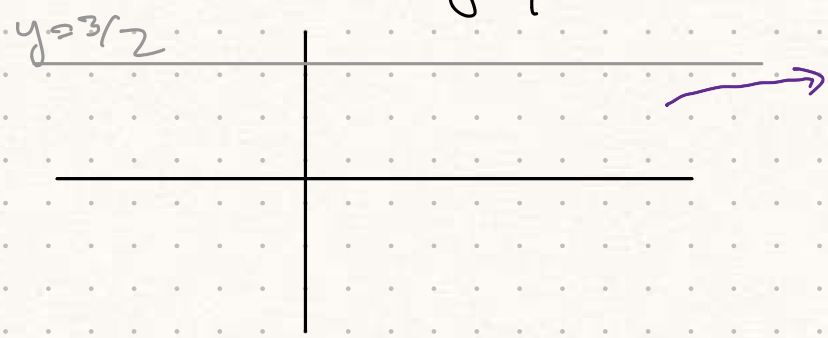
$\lim_{x\to\infty} f(x) = -\infty$

constant.

② finite limit L: $\lim_{x\to\infty} f(x) = L$

$(L \neq \infty)$

In this case we say f has a <u>horizontal asymptote</u> at $y = L$



③ DNE e.g. oscillating.

Example

Does
$$f(x) = \frac{3x-2}{(4x^2+5)^{1/2}}$$

have horizontal asymptote?

$(ab)^{1/2} = a^{1/2} b^{1/2}$

$(x^2)^{1/2} = |x|$

$((-3)^2)^{1/2} = 3$

Positive direction:
$$\lim_{x\to\infty} \frac{3x-2}{(4x^2+5)^{1/2}} = \lim_{x\to\infty} \frac{3x}{(4x^2)^{1/2}}$$

$$= \lim_{x\to\infty} \frac{3x}{2x} = \frac{3}{2}$$

Negative direction:

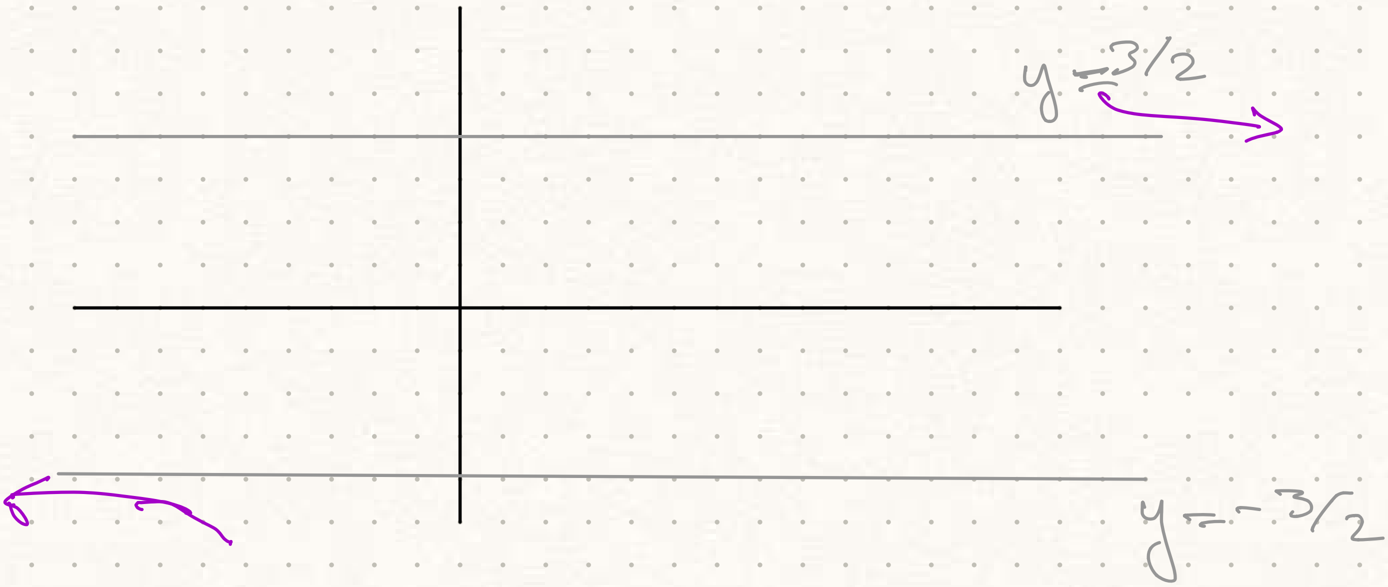
$$\lim_{x\to -\infty} \frac{3x-2}{(4x^2+5)^{1/2}} = \lim_{x\to -\infty} \frac{3x}{(4x^2)^{1/2}}$$

careful!

$$= \lim_{x\to -\infty} \frac{3x}{2|x|}$$

$$= \frac{-3}{2}.$$

There are 2 horizontal asymptotes at $y = 3/2$ and $y = -3/2$.



Example

$$f(x) = \frac{2+3e^x}{7-5e^x}$$

What are the horiz. asymptotes?

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2+3e^x}{7-5e^x}$$

e^x grows quickly as $x \to \infty$.

$$= \lim_{x\to\infty} \frac{3e^x}{-5e^x} = \frac{-3}{5}.$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2+3e^x}{7-5e^x}$$

When $x \to -\infty$

$e^x \to 0$

$(e^{-x} = \frac{1}{e^x})$

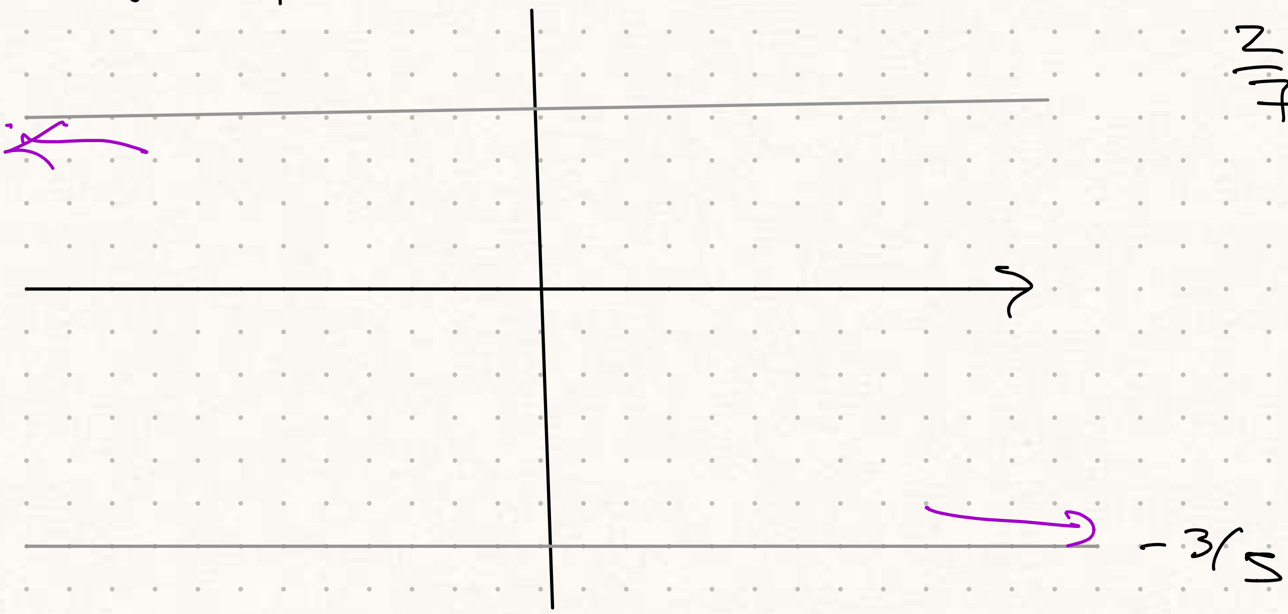
$$= \lim_{x\to-\infty} \frac{2}{7} = \frac{2}{7}.$$

The horizontal asymptotes are

$y = -3/5$

and

$y = \frac{2}{7}$



$$\lim_{x \to -\infty} \frac{3x}{(4x^2)^{1/2}}$$

negative

positive

negative.

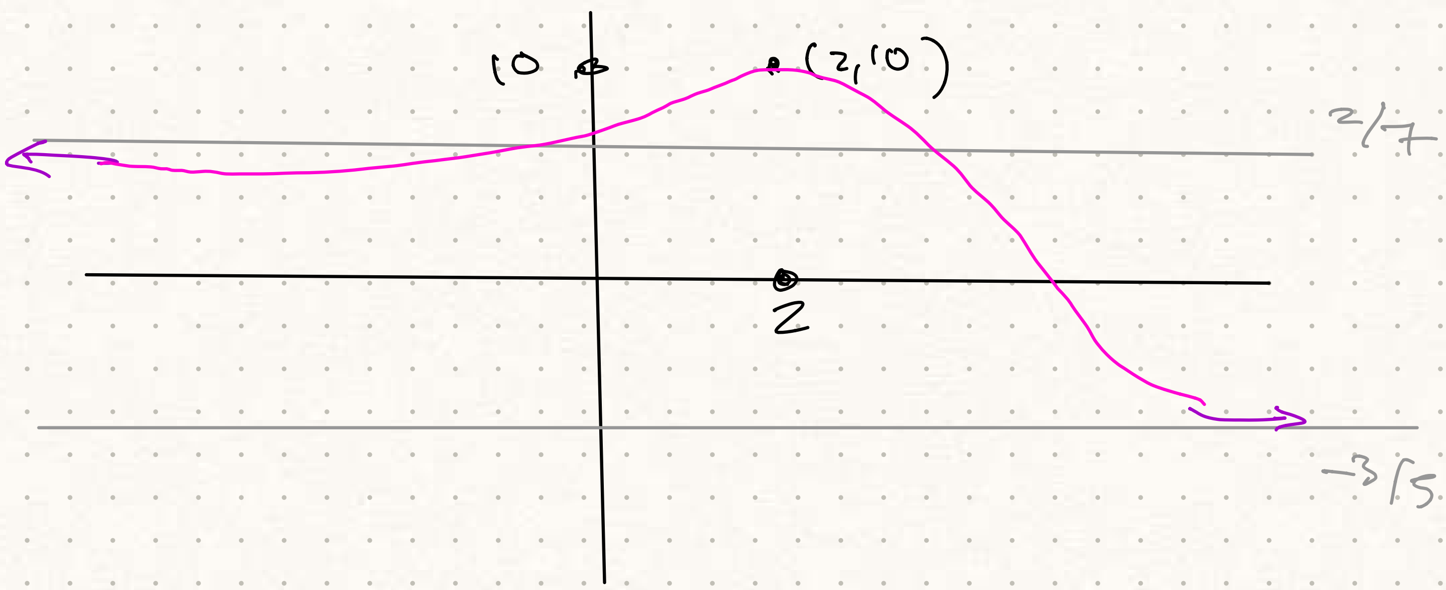
$$\lim_{x \to -\infty} \frac{3x}{4x^2} = 0.$$

Applying this to optimization problems

* Suppose $f(x) = \dfrac{2+3e^x}{7-5e^x}$ has a single critical point at $x = 2$ where $f(2) = 10$. **

Is $x=2$ the abs. max / min / neither?

Answer: Use the "clues" to sketch graph.



From graph, it is obvious that $x=2$ is abs. max.

Again: Knowing critical points

$+$

end behaviour.

allows us to figure out where abs max/min are.

Example

Suppose

* f has critical points at

$x = 0, 1, 2$

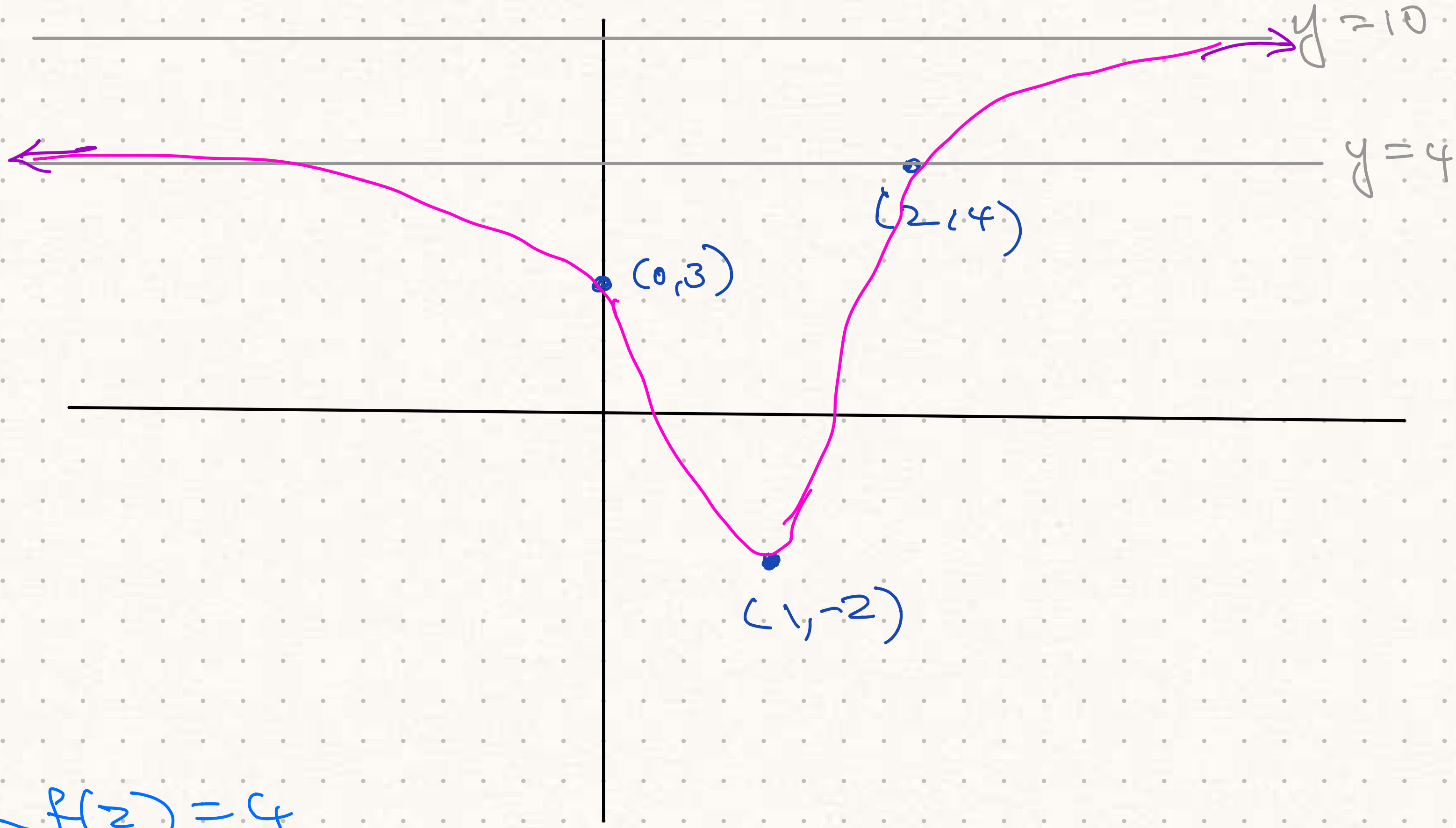
* $f(0) = 3$ $f(1) = -2,$ $f(3) = 4$

* $\lim_{x \to \infty} f(x) = 10$

* $\lim_{x \to -\infty} f(x) = 4.$

What is the abs min/max of f?

Solution: Use the information to sketch a graph



$\leftarrow f(2)=4$

From graph, it is clear that abs min is at $x=1$ $y=-2$

abs max DNE.

Lesson: Easiest way to use information on f, f' is to sketch graph.