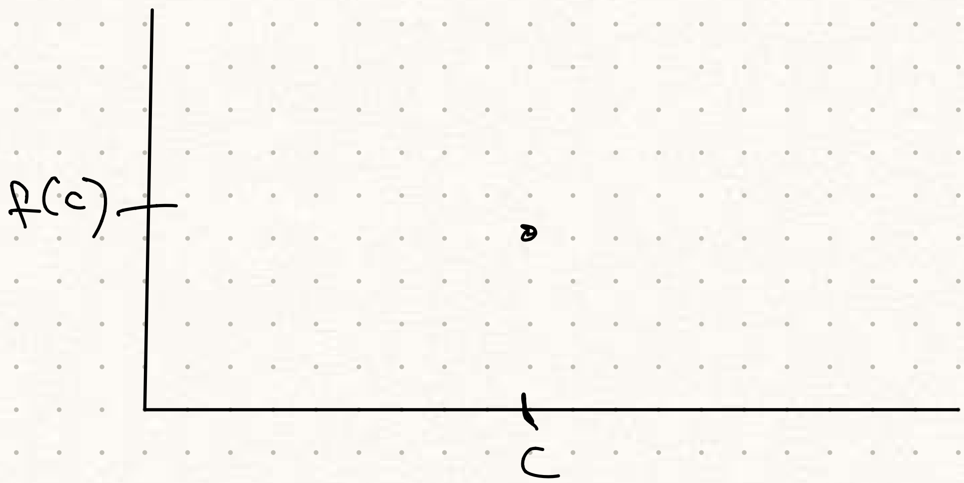
Lecture 34

Recently: optimization problems

Last time we came across this "issue"

if you know:

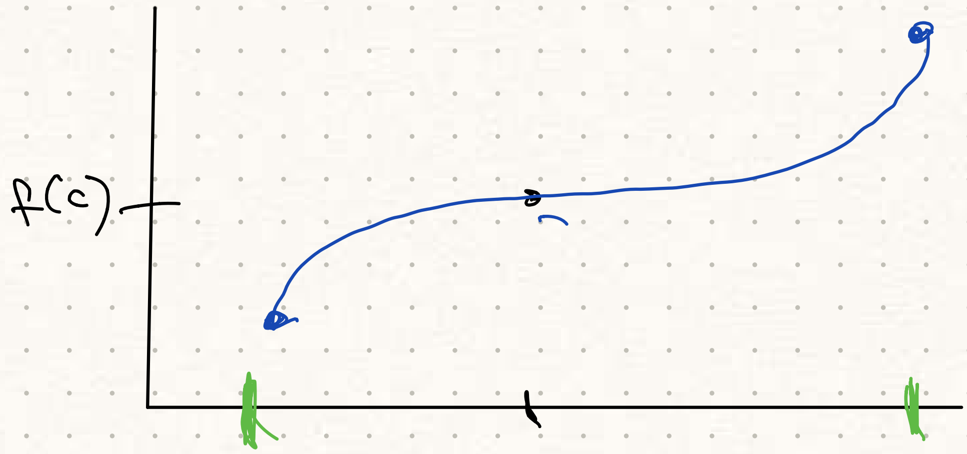
* f only has a critical point at $x=c$, $f'(c)=0$,



How do you know if 'c' is an abs min / abs max / neither

Answer: If domain is bounded, compare $f(c)$ to f at endpoints.

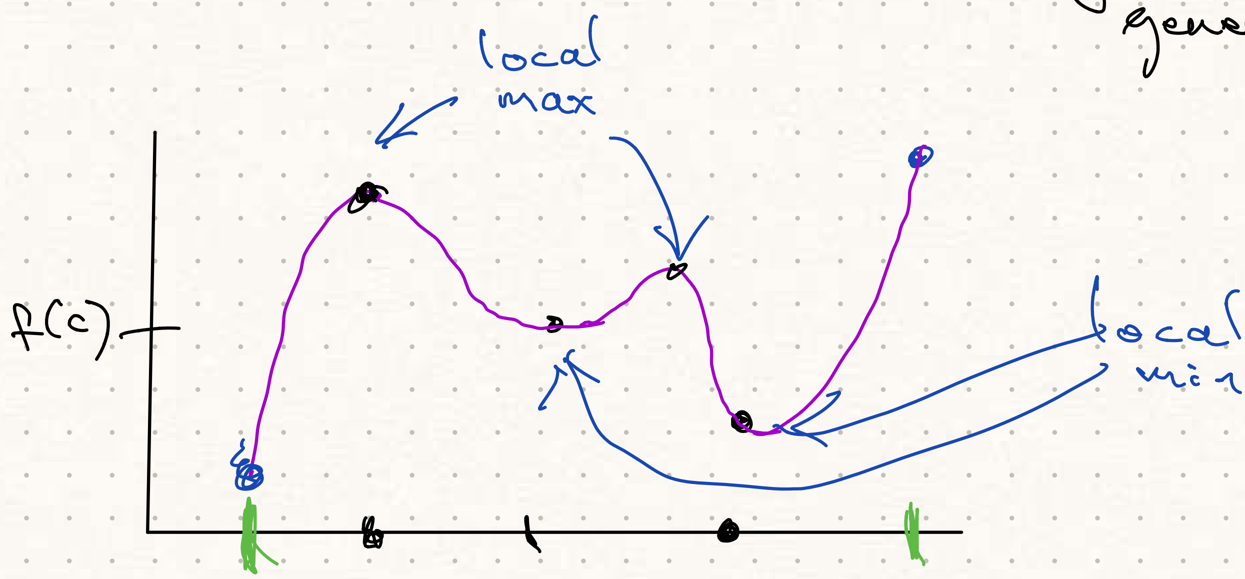
E.g.



y = x

Knowing end behaviour answers our question.

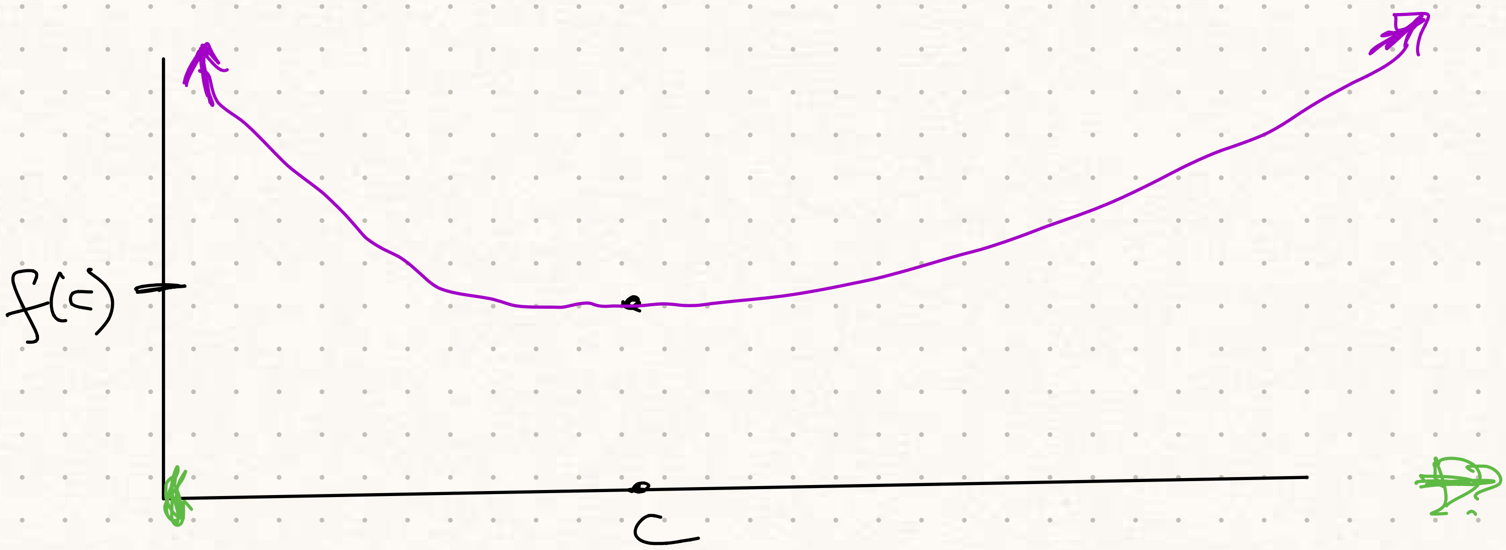
Second derivative test won't be enough, in general.



Second derivative test only tells us if its local min/max

Never enough information to determine abs min/max.

Answer: If domain is unbounded, compare $f(c)$ to $\lim_{x \to \pm\infty} f(x)$



E.g. $\nearrow$ domain $= (0, \infty)$

$$\lim_{x \to 0^+} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = \infty.$$

So graph must look like above.

and c must be abs. min.

Today Ch 4.6

We've already talked about limits as $x \to \infty$, e.g.

$$\lim_{x \to \infty} \frac{1}{x} =$$

a) 0 ← 62%
b) 1
c) 2
d) ∞
e) DNE

Recall

$$\lim_{x \to \infty} f(x) = L$$

means if x is large, $f(x) \approx L$.

$\overset{\text{or}}{=}$ as x gets larger, $f(x)$ gets closer to L.

Thus.

$$\lim_{x \to \infty} \frac{1}{x} = 0$$ because as x gets larger, $\frac{1}{x}$ gets closer to 0

$$\lim_{x \to \infty} \frac{4 + x + 8x^2}{4x^2 + 2} = 2.$$

Why? x^2 dominates x, 4

So we can ignore insignificant terms.

$$\lim_{x \to \infty} \frac{8x^2}{4x^2} = \frac{8}{4} = 2.$$

a) 0
b) 1
c) 2
d) ∞
e) DNE

$$\lim_{x \to \infty} \frac{4 + \sin x + 8x^2}{4x^2} = 2$$

Because $|\sin x| \le 1$ so it's insignificant compared to $8x^2$.

$$\lim_{x \to \infty} \frac{e^x}{4x^2+4} = \infty. \qquad (400(0)).$$

$$\lim_{x \to \infty} e^x = \infty$$

$$\lim_{x \to \infty} 4x^2+4 = \infty.$$

L'Hopital's rule:

$$\lim_{x \to \infty} \frac{e^x}{8x}$$

But

$$\lim_{x \to \infty} e^x = \infty$$

$$\lim_{x \to \infty} 8x = \infty.$$

So L'Hopital's rule:

$$\lim_{x \to \infty} \frac{e^x}{8} = \infty$$

Summary: e^x dominates x

x^2

x^n

$$\lim_{x\to\infty} \frac{e^x + 8x^2 \sin x}{4x^2+4} = \infty.$$

e^x dominates on top.

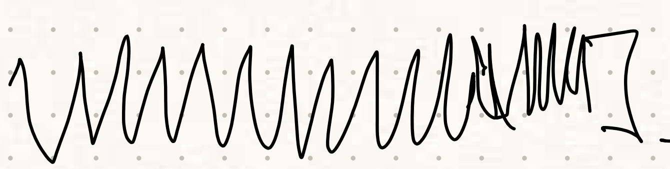
So

$$= \lim_{x\to\infty} \frac{e^x}{4x^2+4} = \infty.$$

$$\lim_{x\to\infty} \sin(x) = L \qquad \text{DNE.}$$

as x gets larger, $\sin(x)$ gets closer to L ?

No way to choose L to make this sentence true.



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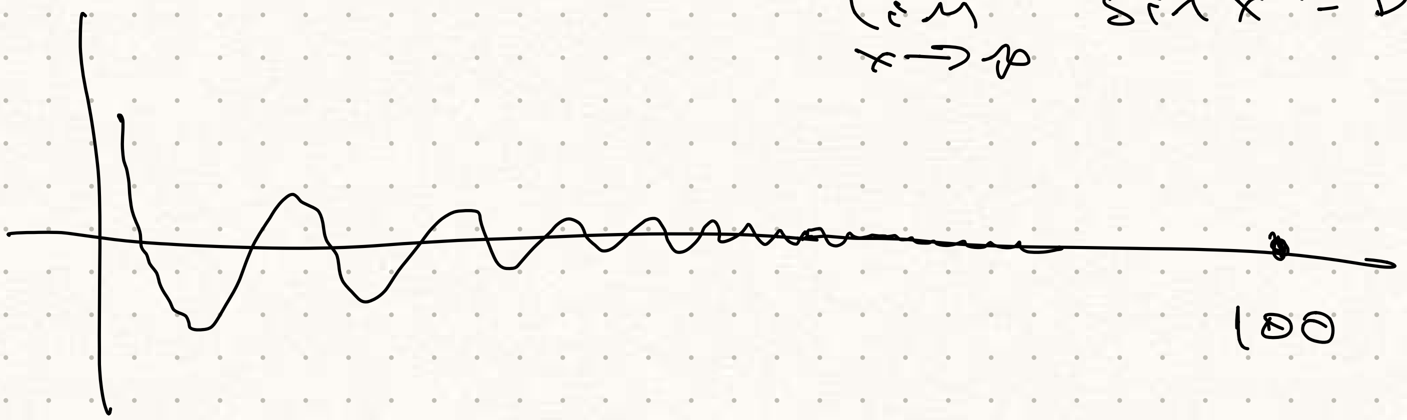
$$\lim_{x\to\infty} \frac{\sin x}{x^2} = 0$$

$$\lim_{x\to\infty} \sin x = DNE$$



$$\lim_{x\to\infty} \frac{3x^2}{2x^2+1} = \lim_{x\to\infty} \frac{3x^2}{2x^2} = \frac{3}{2}.$$

L'hopital's rule

$$\frac{\infty}{\infty} \qquad \frac{0}{0} \qquad \frac{DNE}{\infty}\ ?$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{1/x}{1}$$

$$\lim_{x\to\infty} \frac{\cos x}{2x} = 0.$$

coincidence