Lecture 33

* Midterm graded
  * See gradescope.
  * You can view which rubric items were applied.

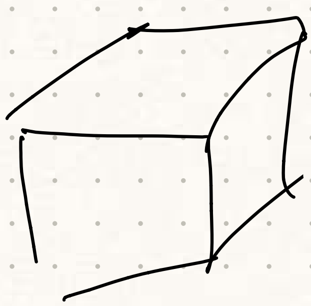
* Grading questions should go to your TA.

More optimization problems Ch4.7

Example
Want Box with volume 216 in^3

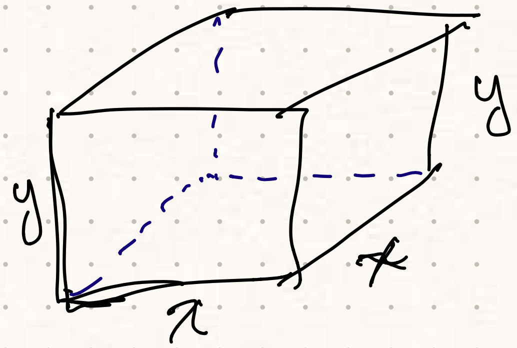
* Open top box
  * (square base)

Want minimize surface area of box



Why? you might be a company making containers and you want to minimize costs.

① Picture / Labels.



② Goal: minimize

$$S = x^2 + 4\underbrace{xy}_{\text{side rectangle}}$$

bottom square

③ Constraint:

$$V = 216 = x \cdot x \cdot y = x^2 y$$

④ Sub constraint into goal

$$216 = x^2 y \Longrightarrow y = \frac{216}{x^2}$$

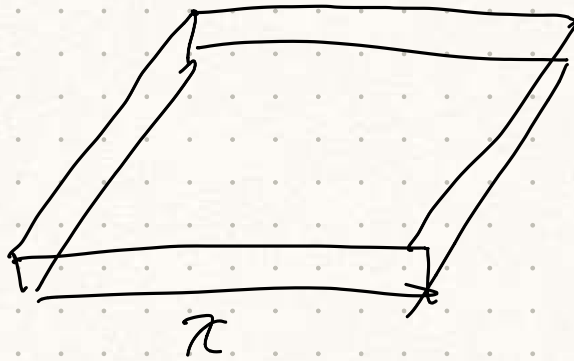
So

$$S = x^2 + 4x\left(\frac{216}{x^2}\right)$$

$$= x^2 + \frac{4 \cdot 216}{x}$$

⑤ Find min

$$S = x^2 + \frac{4 \cdot 216}{x}$$



Domain: $x > 0$

$= (0, \infty)$

unlike other examples, where domain bounded (e.g. $[0, 50]$)

(I) Find candidates:

$$S = x^2 + 4 \cdot 216\, x^{-1}$$

critical points:

$$S' = 0 \Rightarrow 2x - 4 \cdot 216\, x^{-2} = 0$$

$$2x = 4 \cdot 216\, x^{-2}$$

$$x^3 = 2 \cdot 216$$

$$x = (2 \cdot 216)^{1/3}$$

boundary behaviour :

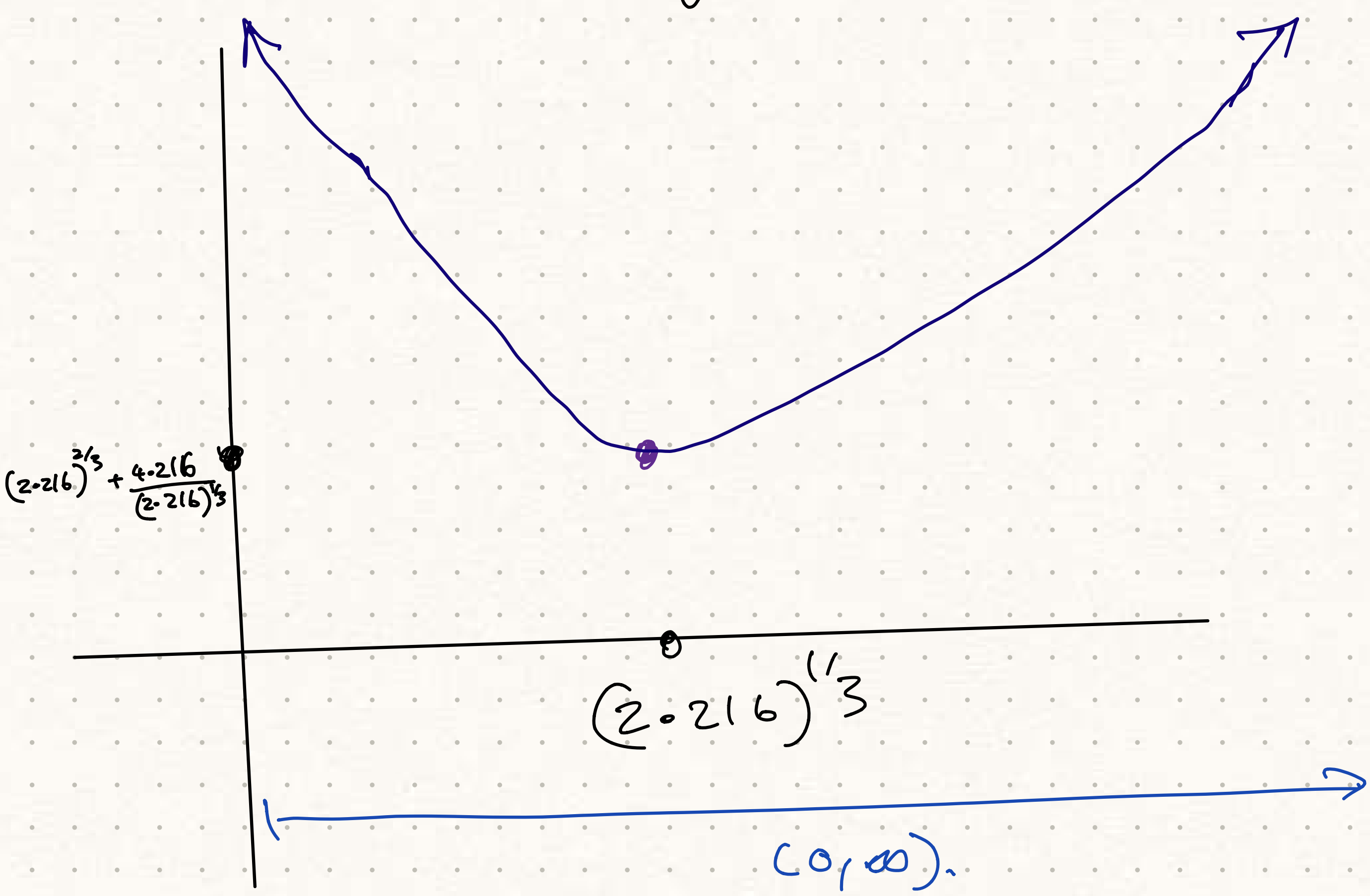
* 0 : $\lim_{x \to 0^+} \left(x^2 + \frac{4 \cdot 216}{x} \right) = \infty$

getting close to boundary.

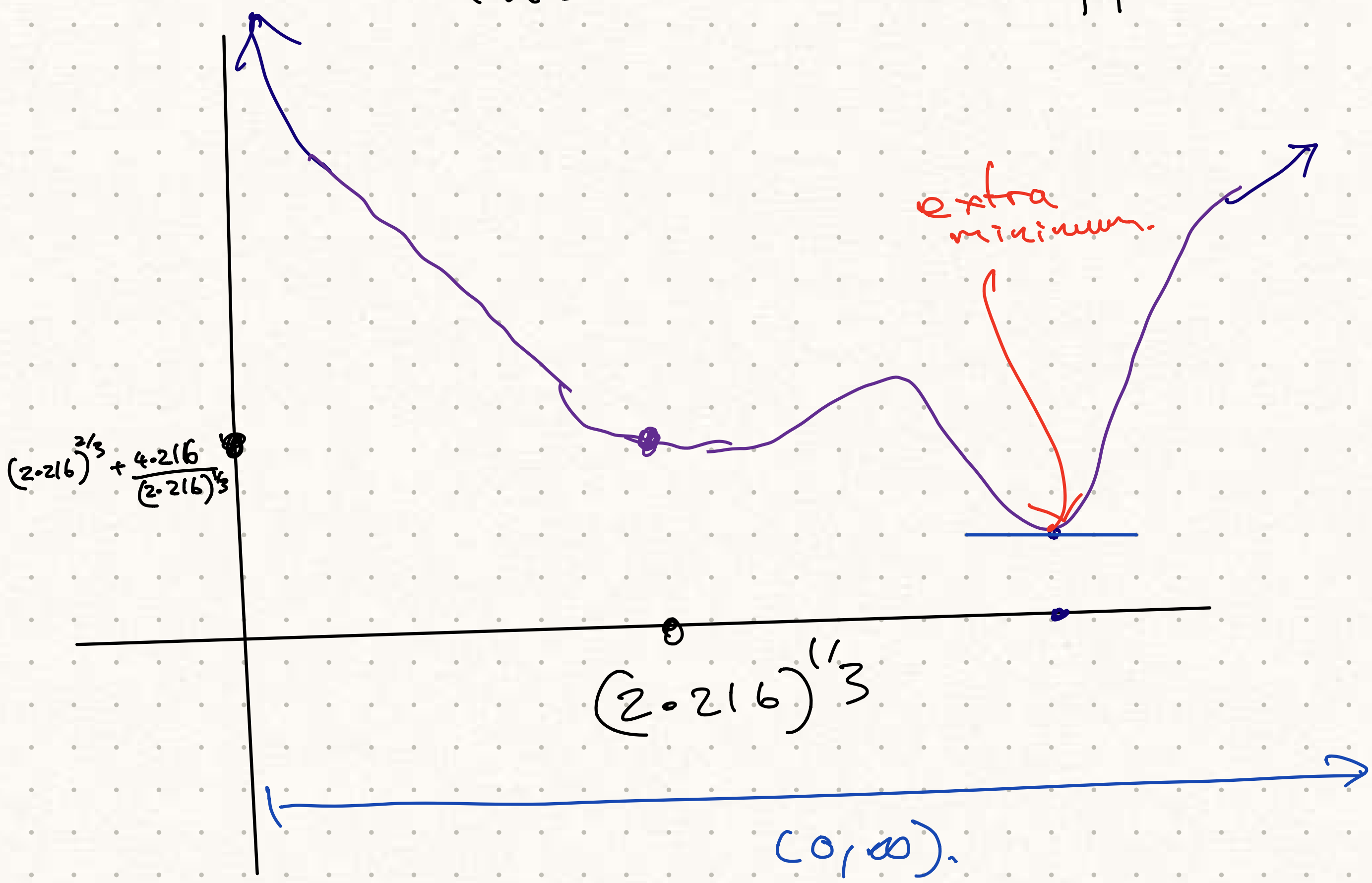
* ∞ : $\lim_{x \to \infty} \left(x^2 + \frac{4 \cdot 216}{x} \right)$

$= \lim_{x \to \infty} x^2 + \lim_{x \to \infty} \frac{4 \cdot 216}{x}$

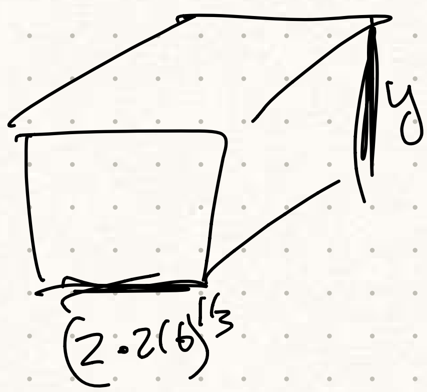
$= \infty + 0$



Q: Why do we know that this doesn't happen?



A: There would be an extra critical points.



Minimum surface area is

$$\boxed{S = (2\cdot216)^{2/3} + \frac{4\cdot216}{(2\cdot216)^{1/3}}}$$

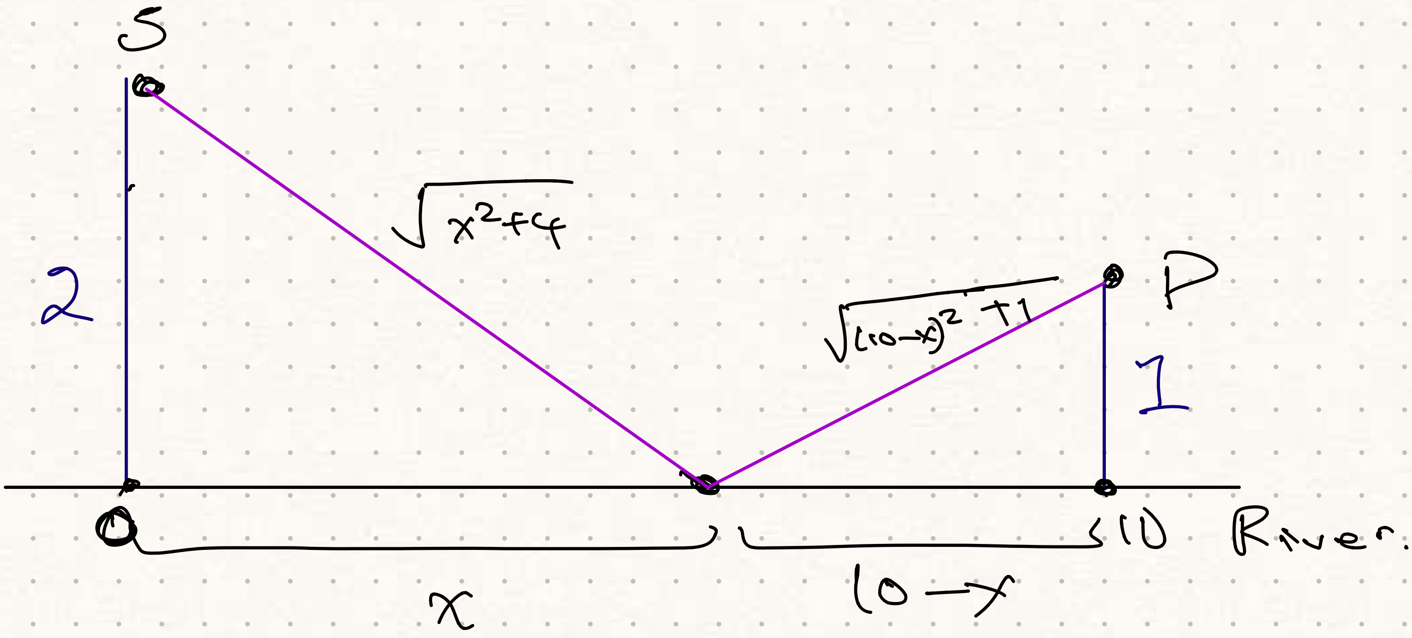
attained at

$$\boxed{\begin{aligned} x &= (2\cdot216)^{1/3} \\ y &= \frac{216}{(2\cdot216)^{2/3}} \end{aligned}}$$

Ex.

A rectangle is inscribed in the ellipse

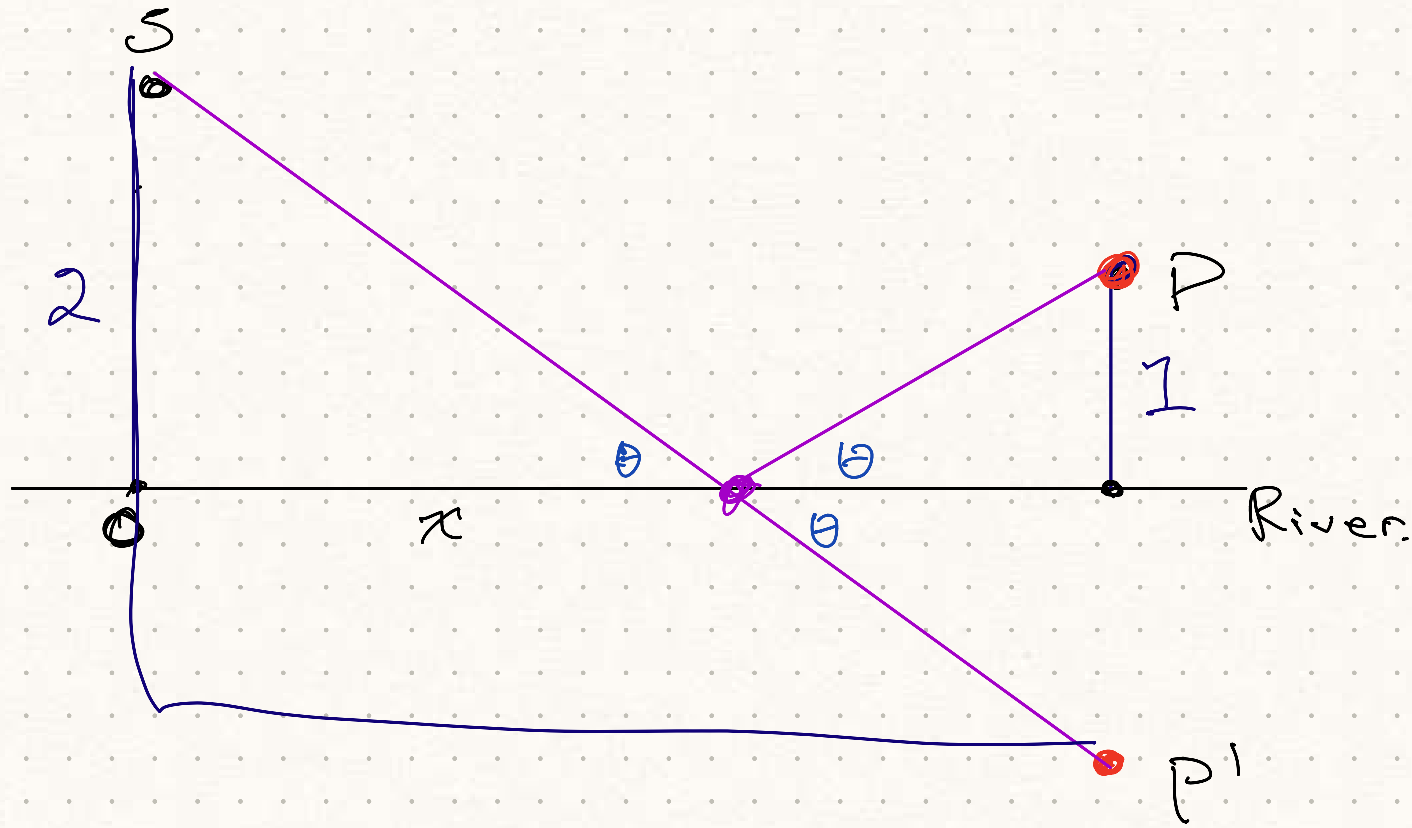
$$\frac{x^2}{4} +$$

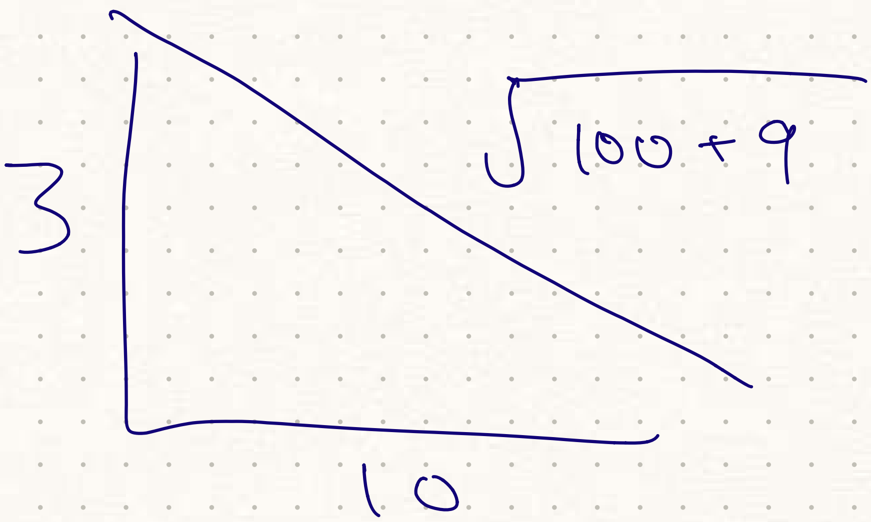


$$\text{dist} = \sqrt{x^2+4} + \sqrt{(10-x)^2+1}$$

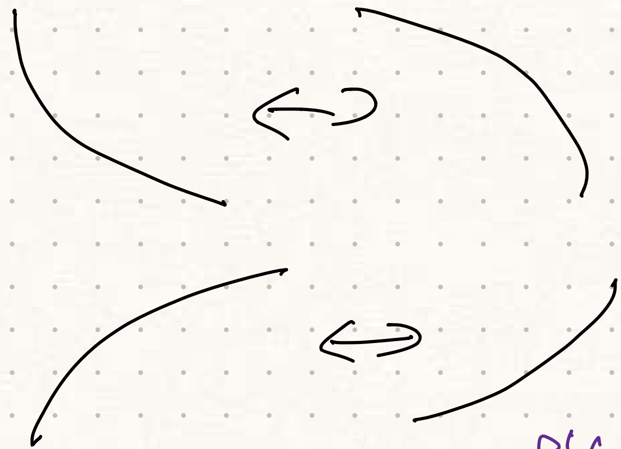
min dist is attained at

$$x = \frac{20}{3} \approx 6.66\ldots$$

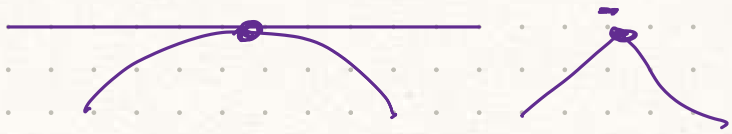

S
2
P
1
θ
θ
θ
O
x
River
P'

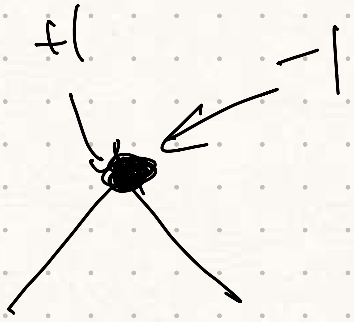

3
√100 + 9
10

$f(x) =$

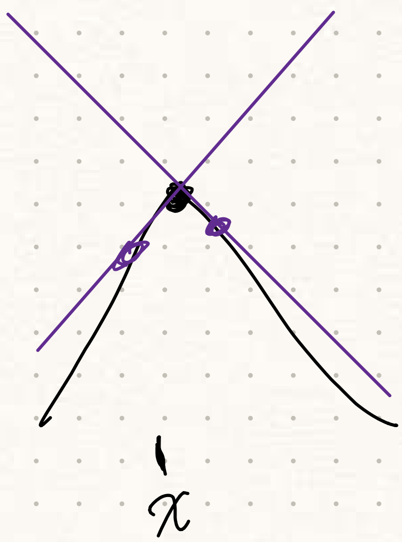


$f'(x) = 0$ $\qquad$ $f'(x)$ DNE





$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$$\frac{0}{1} = 0$$

$$\lim_{x \to 0} \frac{\sin x}{x^2+1} = \frac{\lim_{x \to 0} \sin x}{\lim_{x \to 0} x^2+1}$$

$$\lim_{x \to 0} \frac{\sin x}{x^2+1} = 0$$

$$\cancel{\lim_{x \to 0} \frac{\cos x}{2x} = \frac{1}{0}}$$

$$\lim_{x \to \infty} \frac{\ln x}{x}$$

$$\lim_{x \to \infty} \frac{1}{x}$$