

## Lecture 32

\* Midterm grades next Monday.

Plan for rest of the semester.

\* Finish Ch 4

Quiz Find maximum of

$$f(x) = 50x - x^2 \quad \text{on } [0, 50].$$

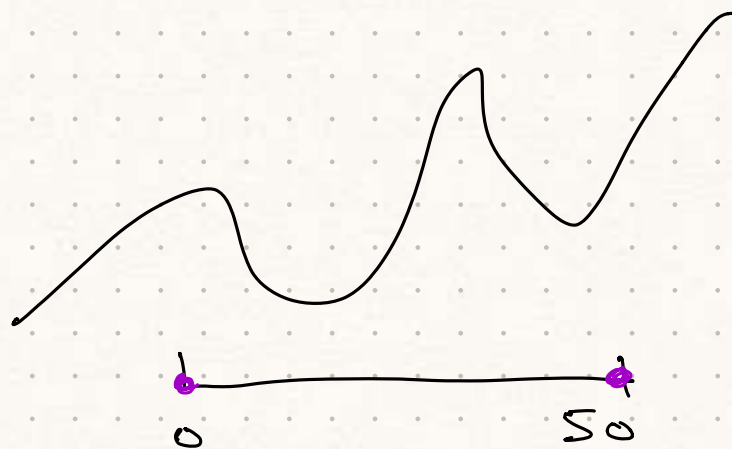
a) 825

b) 250

c) 375

d) 0

e) 625 ← 35%.



Recall the steps.

1) Find candidates:

a) Endpoints  $x=0$ ,  $x=50$

b) critical points:  $f'(x)=0$  (or undef)

2) Plug candidates into  $f$  to find biggest.

1) Find candidates:

- a) Endpoints  $x=0$ ,  $x=50$  (or undef)
- b) critical points:  $f'(x)=0$

$$f'(x) = 50 - 2x$$

crit point at  $x=25$

2) Plug candidates into  $f$  to find biggest.

$x$	$50x - x^2 = x(50 - x)$
0	0
50	0
25	$25^2 = 625$

Ch 4.7

Today.

# Applied optimization.



Finding "best"  
i.e. Finding biggest/smallest.

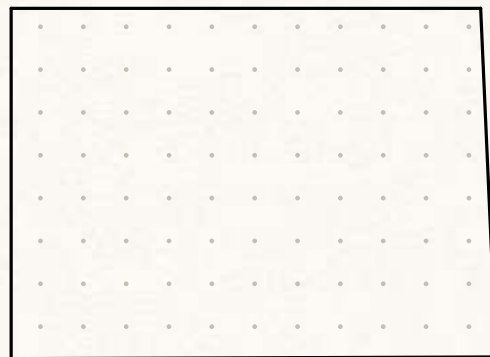
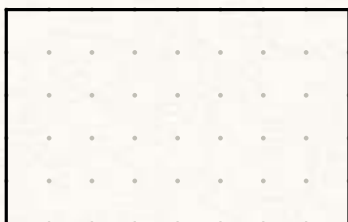
## Optimization

General problem

- \* Want to maximize (or min) something.
- \* But we have constraints.

E.g.

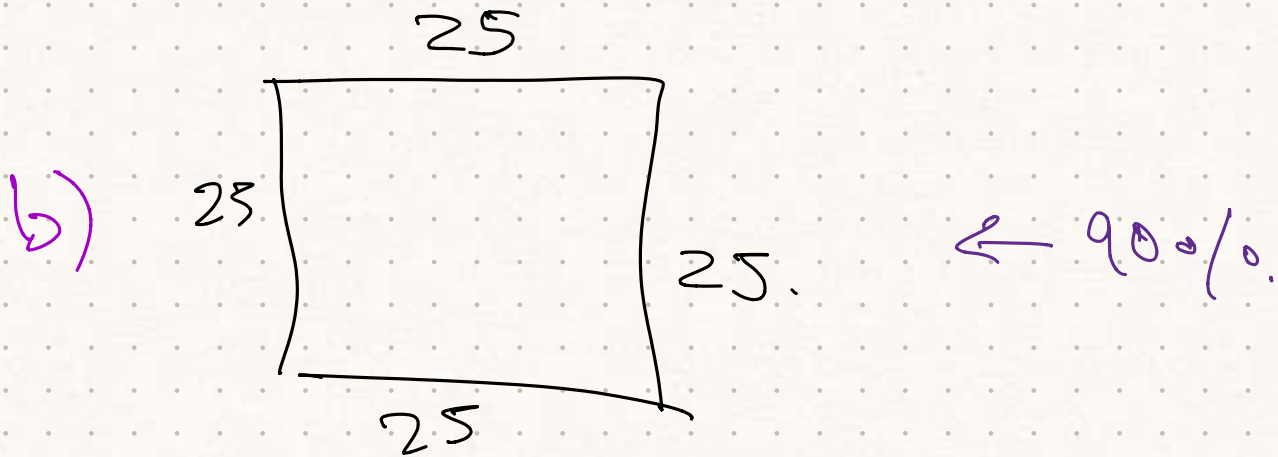
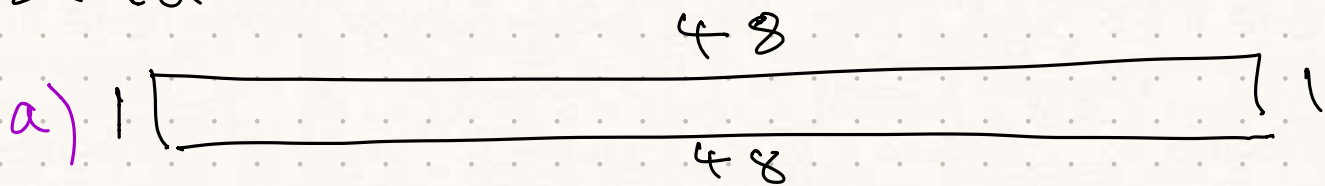
- \* Make a rectangular garden have biggest area possible.



Not interesting unless we add constraints.

\* Suppose we only have 100ft of fencing.

Should I do:

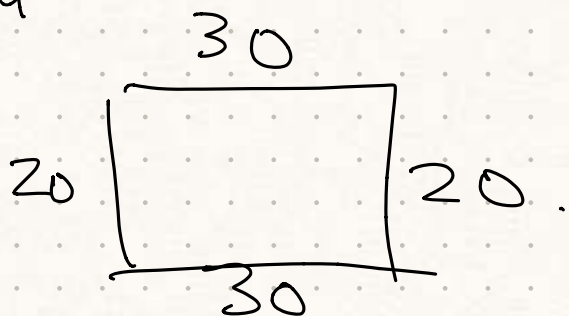


Which one has bigger area?

a)  $\text{Area} = 48 \cdot 1 = 48.$

b)  $\text{Area} = 25 \cdot 25 = 625.$

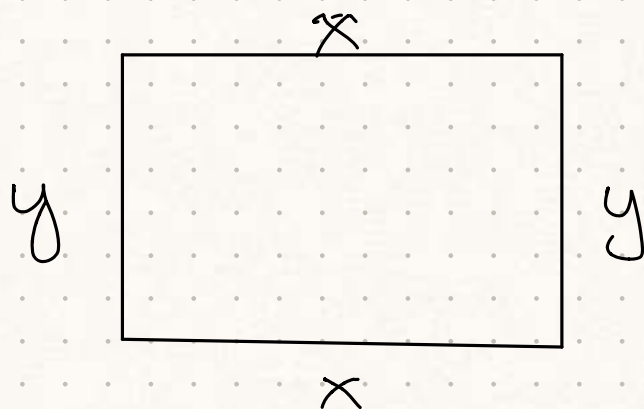
What about



Calculus / derivatives allows us to find max without testing all possibilities

Soln (turning a application into computation).

① Draw picture / label quantities



② Write down goal in terms of variables:

Maximize  $A = x \cdot y$

③ Write down constraint (in terms of variables)

$$x + x + y + y = 2x + 2y = 100.$$

④ Substitute constraint into "goal",  
to get function of only  
one variable.

$$2x + 2y = 100$$

$$\text{so } 2y = 100 - 2x$$

$$y = 50 - x.$$

so

$$A = xy = x(50 - x)$$

write domain:

$$x \geq 0$$

$$x \leq 50$$

5). Maximize goal, on domain.

$$\text{Maximize } x(50 - x) = 50x - x^2$$

on  $[0, 50]$ .

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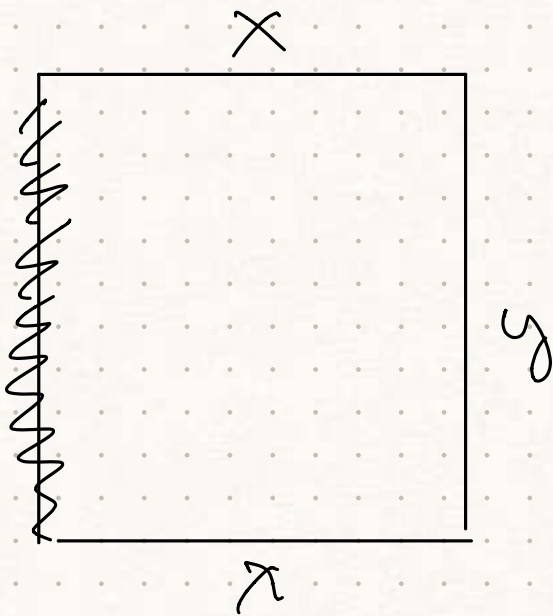
Did this in quiz: max  
at  $x=25$ ,  $A=625$ .

Example \* Make rect as big as possible

\* 100ft of fence

\* One side of garden is rock wall, doesn't need fence.

①



② Goal: Maximize  $A = x \cdot y$

③ Constraint:  $2x + y = 100$

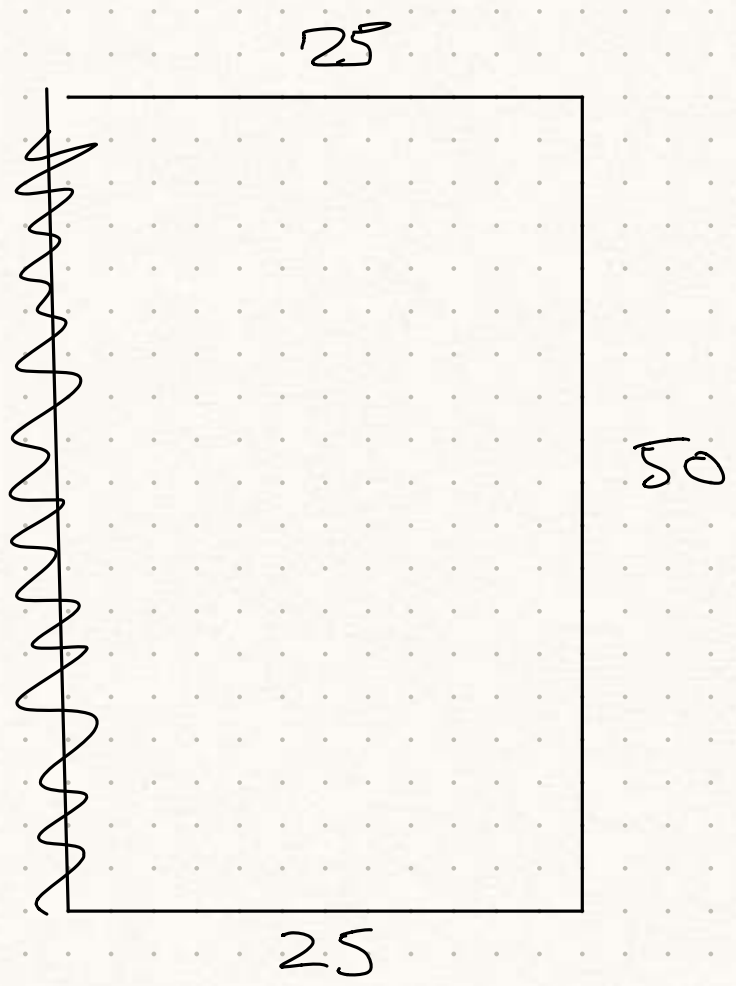
④ Sub. constraint into goal:  
 $2x + y = 100 \Rightarrow y = 100 - 2x$

so  $A = x(100 - 2x)$

5). Maximize

$$A = x(100 - 2x) \quad \left| \begin{array}{l} x \geq 0 \\ x \leq 50 \end{array} \right. \quad \text{(otherwise)}$$

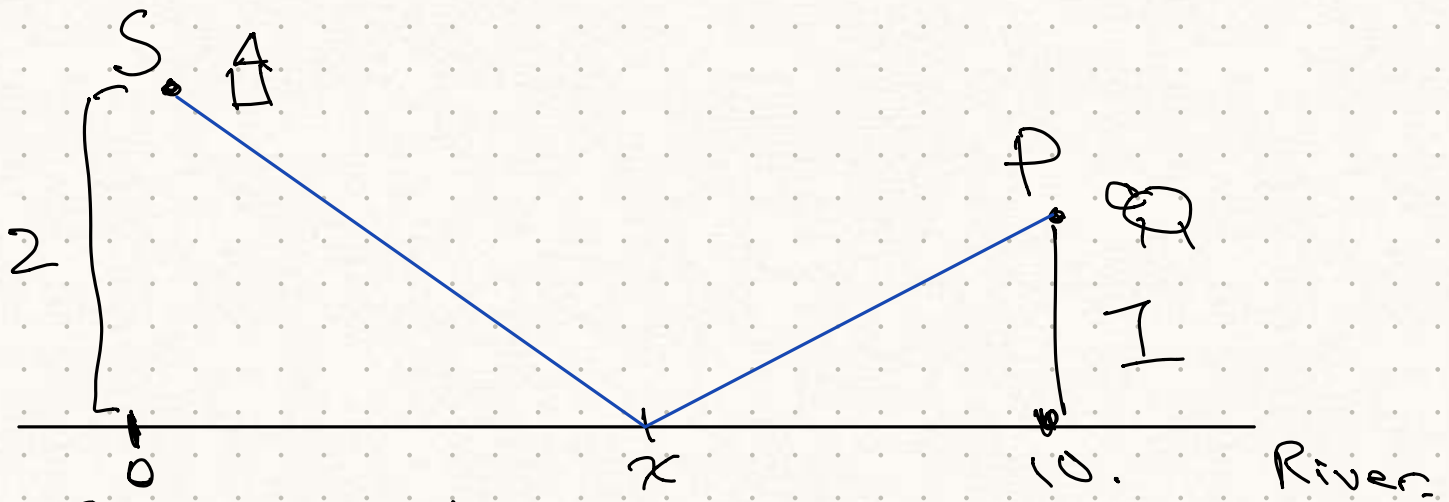
Using procedure in quiz at start:  
Max at  $x = 25$   
 $y = 100 - 2x = 100 - 50 = 50$   
 $A = 1250$





# Example

①



S needs to go to P,

but needs to touch River  
at some point.

Best path?

What point on the  
river should she  
aim towards?

~~①~~ ② Maximize

$$d = \sqrt{x^2 + 2^2} + \sqrt{(10-x)^2 + 1^2}$$

Use procedure in quiz to  
find optimal  $x$ .  
(Soln:  $x = \frac{20}{3}$ ) Exercise.