Lecture 31

Exam prep:

* Do the midterm "Practice Submission", to submit to gradescope.com. By 10pm tonight.

  You need
  * phone (camera)
  * paper + pencil.

* You should have email from gradescope. (after lecture).

* Video on how to submit in chat.

* See blackboard for detailed instructions + practice submissions

* It's your responsibility to make sure you can submit!

Mat 125 Review sesion

5:30pm – 7 pm Tuesday, Zoom

Link will be posted on bb, piazza, my website.

Will be recorded.

Last few lectures:

* applications of derivatives:
  * Tangent line / linear approximation
  * Using derivative to find min/max
  * Using deriv. to sketch graph.

Today: Ch 4.8 L'Hopitâls Rule

Using derivatives to evaluate limits.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Eval limit

Compute deriv.

L'Hopital's rule (today).

Recall

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)},$$ if f, g cts and $g(a) \neq 0$.

E.g. $\lim_{x\to 2} \frac{x^2-4}{x-1} = \frac{4-4}{2-1} = 0.$

Motivating Examples

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

can't plug in 2, get $\frac{0}{0}$.

$$= \lim_{x\to 2} x+2$$

$$= 4.$$

L'Hopital's rule:

if $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Applying this to simple example

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{2x}{1} = 4$$

Example

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = \cos(0) = 1.$$

Why does it work?

(Idea: use linear approximation).

Suppose $\lim_{x \to a} f(x) = 0$, $\lim_{x \to a} g(x) = 0$

then
$$\lim_{x \to a} \frac{f(x)}{g(x)} \approx \lim_{x \to a} \frac{f(a) + f'(a)(x-a)}{g(a) + g'(a)(x-a)}$$

$$= \lim_{x \to a} \frac{f'(a)\cancel{(x-a)}}{g'(a)\cancel{(x-a)}}$$

$$= \frac{f'(a)}{g'(a)}.$$

(L'Hopitals works for many indeterminate forms, such as

$$\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad \infty^0, \quad 0^\infty.$$

<u>Example</u>

$$\lim_{x\to\infty} \frac{3x+5}{2x+5} = \lim_{x\to\infty} \frac{3}{2} = \frac{3}{2}.$$

<u>Example</u>

$$\lim_{x\to\infty} \frac{\ln x}{5x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{5} = \lim_{x\to\infty} \frac{1}{5x}$$

$$= 0.$$

<u>Warning:</u> Only works if the form is indeterminate.

E.g. $\lim_{x\to 1} \frac{x^2+5}{3x+4} = \frac{6}{7}.$

$$\lim_{x\to 1} \frac{x^2+5}{3x+4} = \lim_{x\to 1} \frac{2x}{3} = \frac{2}{3}.$$

WRONG.

What went wrong?

L'hopital's rule does not apply because $\lim_{x\to 1} x^2+5 \neq 0.$

$\lim_{x\to 1} 3x+4 \neq 0.$

---

* Try to do it by direct sub.

if you don't get $0/0$, $\frac{\infty}{\infty}$, etc, done.

if you do, use L'hopitals.

Example:

$$\lim_{x\to 0^+} x\cdot\ln(x) = 0\cdot(-\infty).$$

Use L'hopitals:

$$\lim_{x\to 0^+} x\cdot\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{-x^2}{x}$$

L'hopital

$$= \lim_{x\to 0^+} -x.$$

$$= 0$$

---

multiplying by x on numerator/denom

Explanation: $\frac{\ln x}{\frac{1}{x}} = \frac{x\cdot\ln x}{x\cdot\frac{1}{x}} = \frac{x\cdot\ln x}{1}.$

Explanation: $\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = -x^{-2} = \frac{-1}{x^2}.$

Example

$$\lim_{x \to 0^+} \frac{1}{x} - \frac{1}{2x} = \lim_{x \to 0^+} \frac{2x - x}{2x^2}$$

$$= \lim_{x \to 0^+} \frac{x}{2x^2}$$

$$= \lim_{x \to 0^+} \frac{1}{2x} = \infty.$$

Example

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right).$$

$\infty - \infty$

$$= \lim_{x \to 0^+} \left(\frac{\sin x}{x \sin x} - \frac{x}{x \sin x} \right)$$

$$= \lim_{x \to 0^+} \left(\frac{\sin x - x}{x \sin x} \right)$$

L'hopital:

$$= \lim_{x \to 0^+} \frac{\cos x - 1}{\sin x + x \cos x}$$

Use L'hopital again:

$$= \lim_{x \to 0^+} \frac{-\sin x}{\cos x + \cos x - x \sin x} = \frac{0}{2} = 0.$$

$x \cdot \ln x$

$\frac{f}{g}$

$x^2 - 2x$

$2x - 2$

$f' < 0$
$f' > 0$

$\times$

$f'' < 0$
$f'' > 0$

$f = P_{bike} - P_{car}$

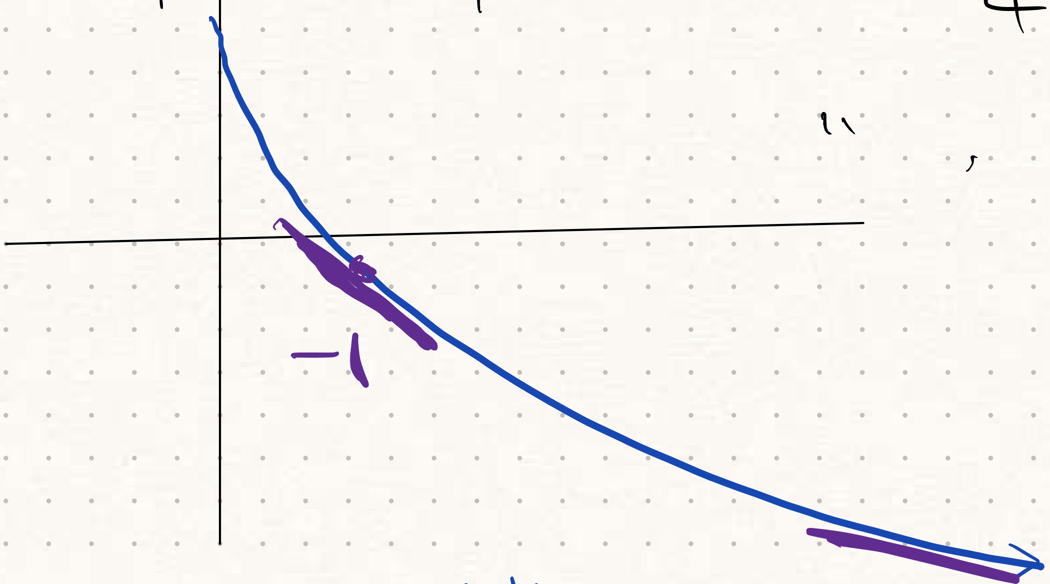
$f' = P_{bike}' - P_{car}'$

$f'' = P_{bike}'' - P_{car}'' > 0$

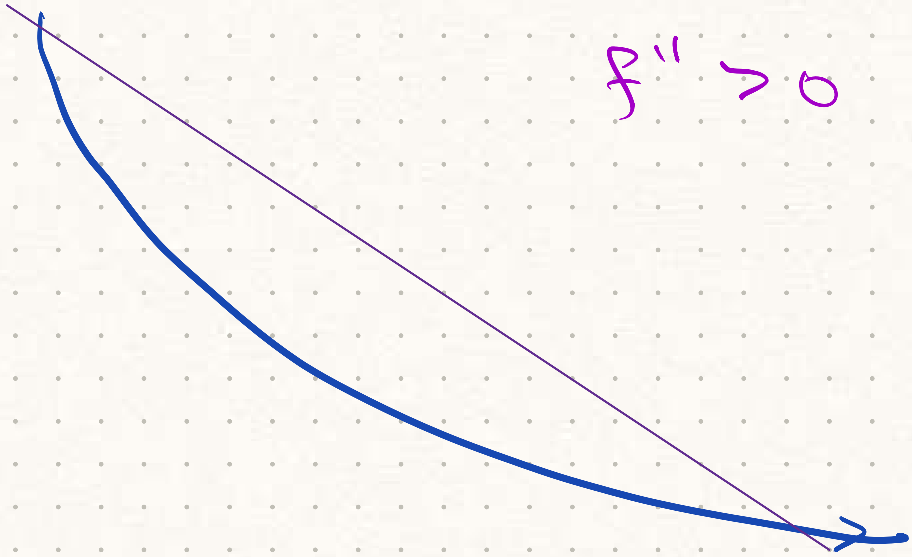
$P_{bike}'' > P_{car}''$

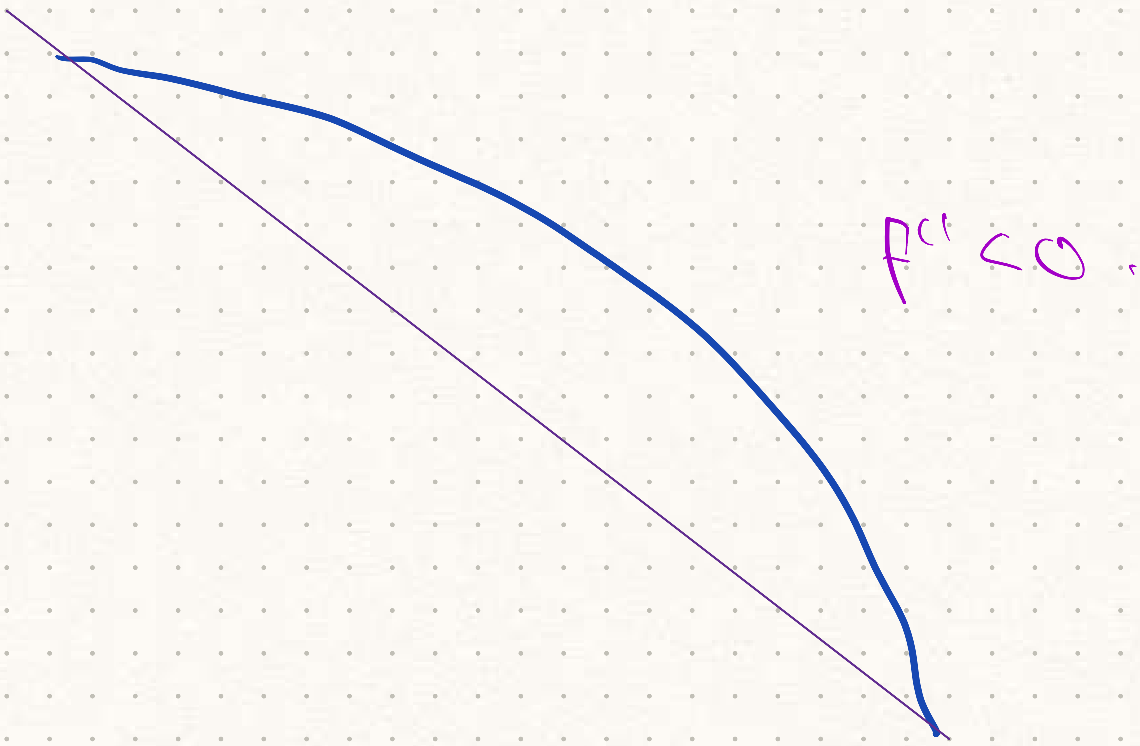
$f'(x) = P_{bike}'' - P_{car}$

4 combo of



$f'' > 0$


f'' > 0


f'' < 0