Lecture 30

* Information about exam:

  April 22 8:30 – 10:15.

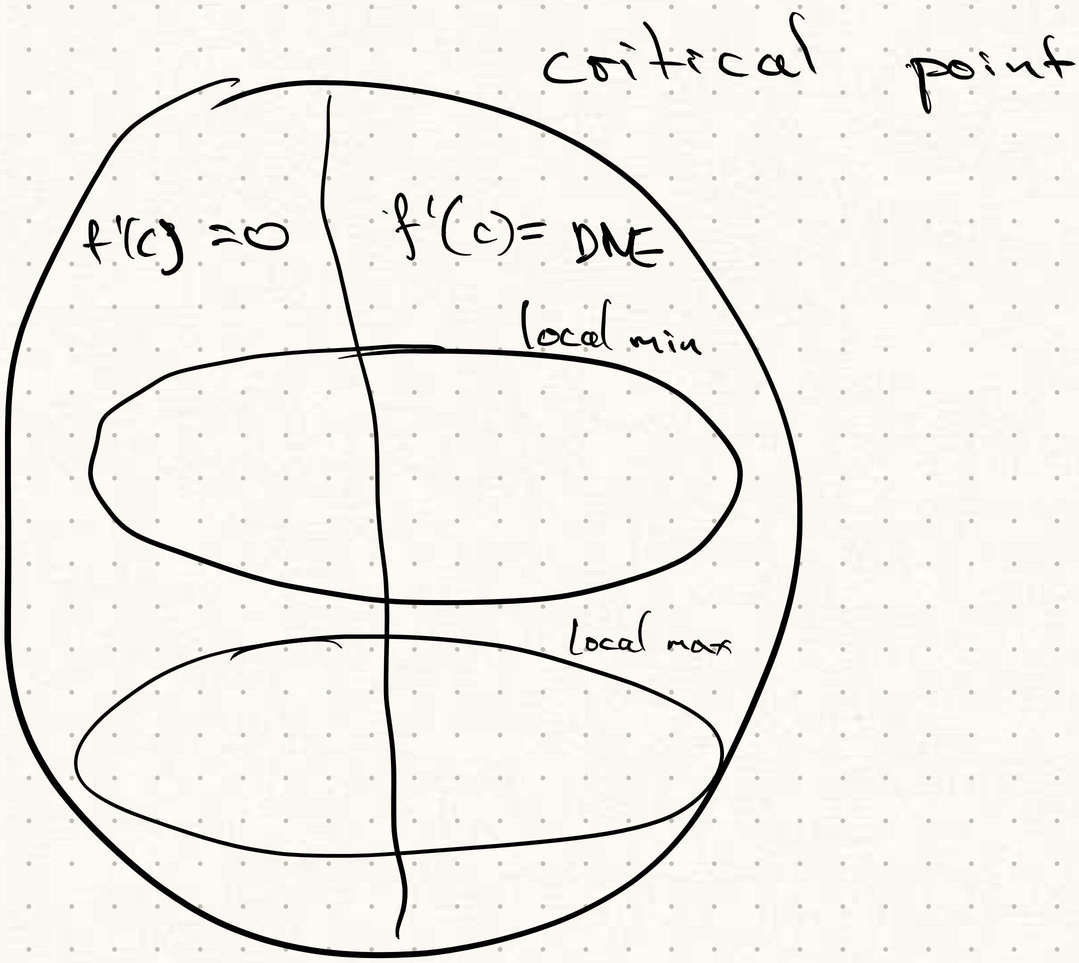
* Do work on paper and then take a photo, then upload to gradescope.com.
  either pdf or images.

* Demostration:

* Practice exam been posted.

* Official instructions to come from Prof. Andersen.

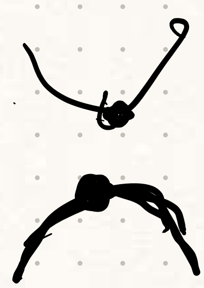
Last few lectures.



Venn diagram.

local min : "valley" :

local max : "peak"



Should be able to explain why these relationships hold.

Today: Finish 4.5.

Key fact:

$f' > 0$ means f is increasing ("uphill")

$(\nearrow)$

$f' < 0$ means f is decreasing ("downhill").

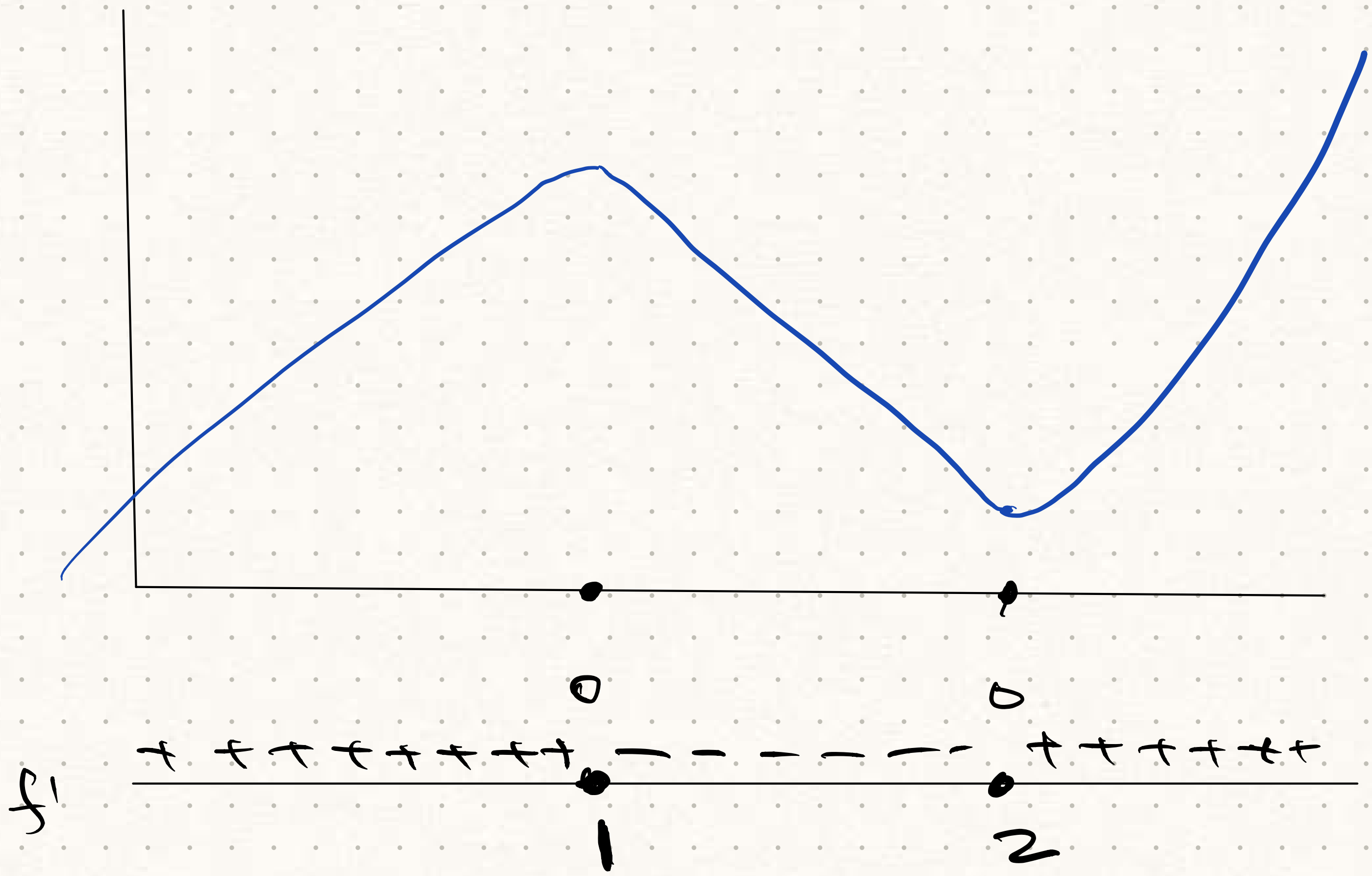
$(\searrow)$

So, if you know the signs of f', you can sketch a graph of f.

E.g. Suppose

$$\begin{cases} f' > 0 & \text{on } (-\infty, 1) \\ f'(1) = 0 & \\ f' < 0 & \text{on } (1, 2) \\ f'(2) = 0 & \\ f' > 0 & \text{on } (2, \infty) \end{cases}$$

Sketch what the function looks like.



From this "technique" we see that if $f'(c)=0$, we can determine whether it is

* local max
* local min
* neither ~~& inflection.~~

First derivative test

if c is a critical point of f,

1) if f' changes from pos to neg, then c is local max.

2) if f' changes from neg to pos, then c is local min.

3) if f' doesn't change sign c is neither local min or local max.

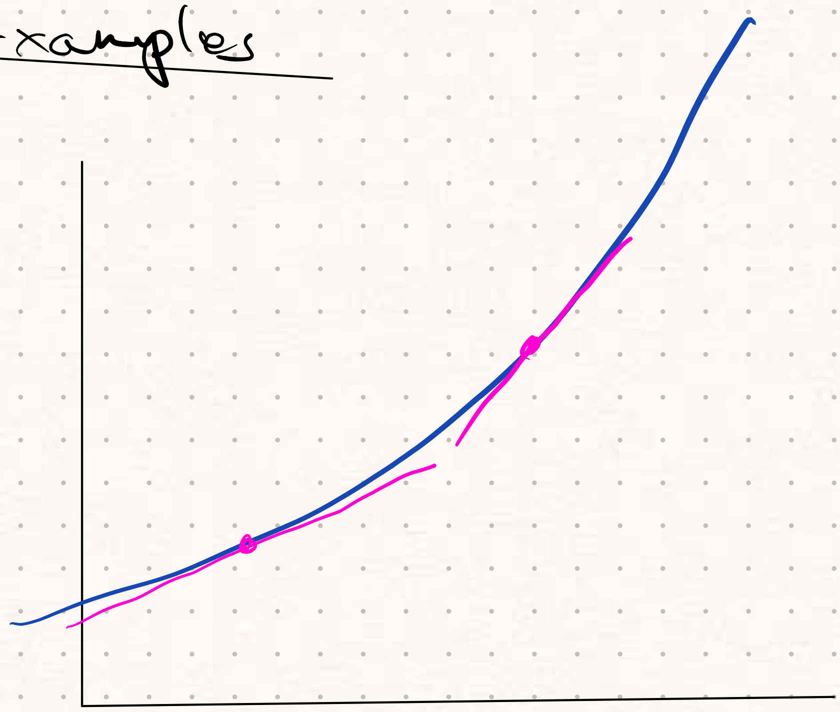
Concavity

Definition: if f' is increasing

$\Leftrightarrow$ f is concave up.

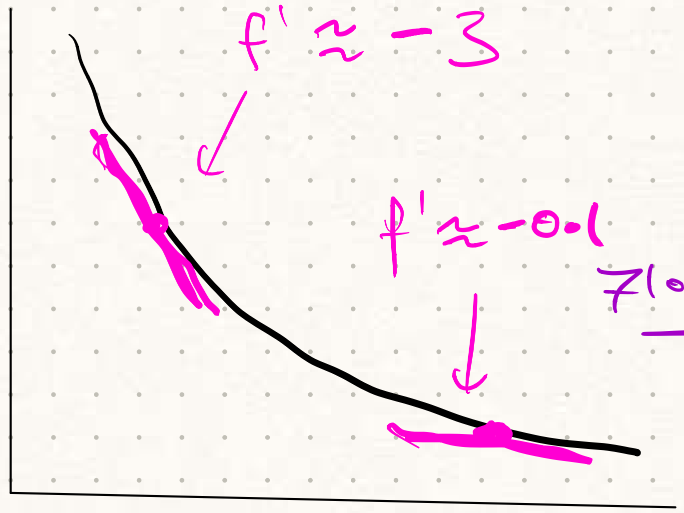
f' decreasing, f is concave down.

Examples



f' increasing so f is concave up.

Quiz



71%

f is

a) concave up

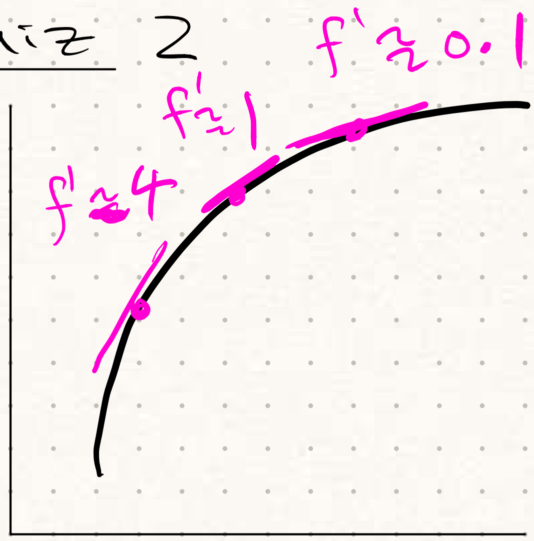
b) concave down.

c) neither.

$-0.1 > -3$

So f' increasing, so f is concave up.

Quiz 2



65%

f is

a) concave up

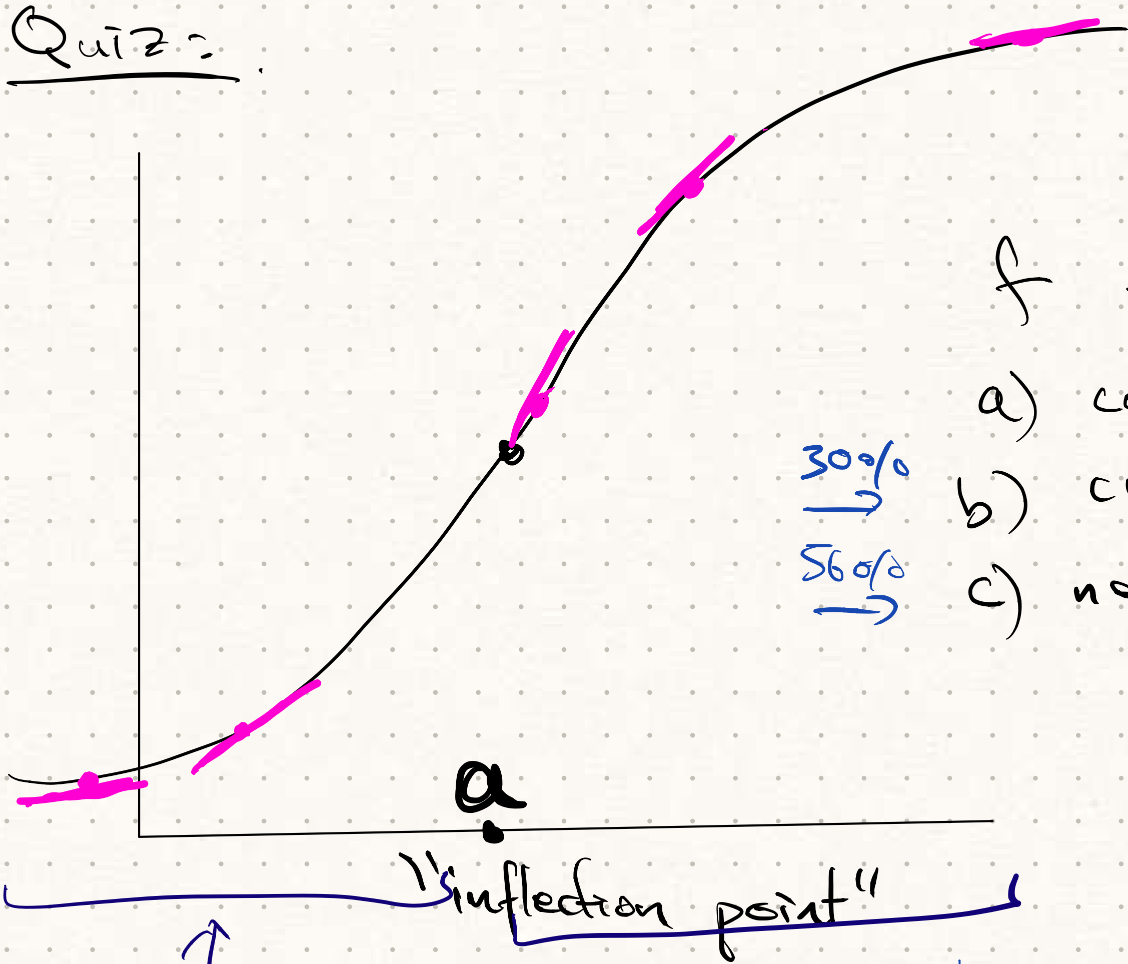
b) concave down

c) neither.

f' is decreasing

f is concave down.

Quiz:



f is

30% → a) concave up

56% → b) concave down.

c) neither.

f' increasing

so

concave up.

on

$(-\infty, a)$

f' decreasing

so concave down.

on

(a, ∞)

Note: f itself

is increasing

always.

You have already heard "inflection point" in relation to the number of coronavirus cases.

Observation

Concave up $\iff$ f' increasing $\iff$ $f'' > 0$

Concave down $\iff$ f' decreasing $\iff$ $f'' < 0$.

So now we can answer:

$$f(x) = x^3 - 6x^2 + 9x + 30$$

where is it concave up?
it concave down?

Soln:

$$f'(x) = 3x^2 - 12x + 9$$

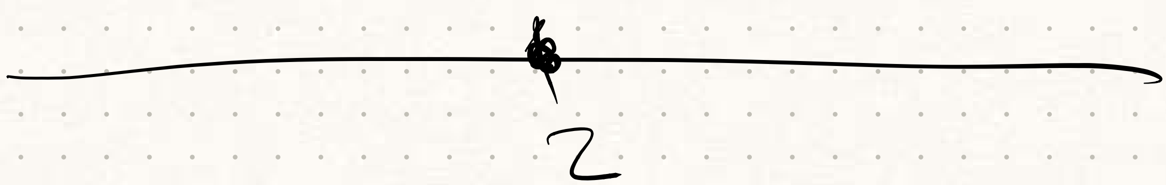
$$f''(x) = 6x - 12$$

$$\text{So } \begin{cases} f''(x) > 0 & \text{if } x > 2 \\ f''(x) < 0 & \text{if } x < 2. \\ f''(x) = 0 & \text{if } x = 2. \end{cases}$$

f is concave up on $(2, \infty)$.

f is concave down on $(-\infty, 2)$.

You can combine this with the first derivative to get sketch of f.



$$f'(x) = 3x^2 - 12x + 9.$$
$$= 3(x^2 - 4x + 3)$$
$$= 3(x-4)(x+1).$$

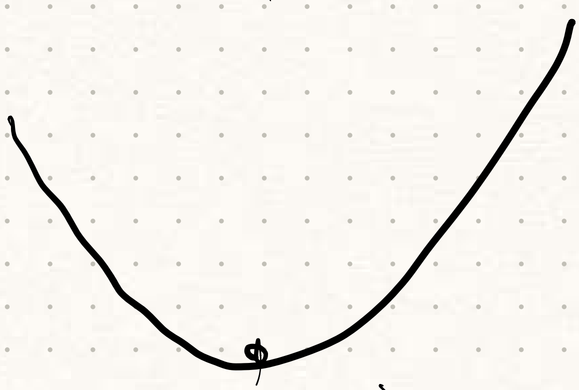
So $x = 4$ and $x = -1$ are critical points.

$f''(4) = 0$ positive, so 4

$f''(-1) = -18.$

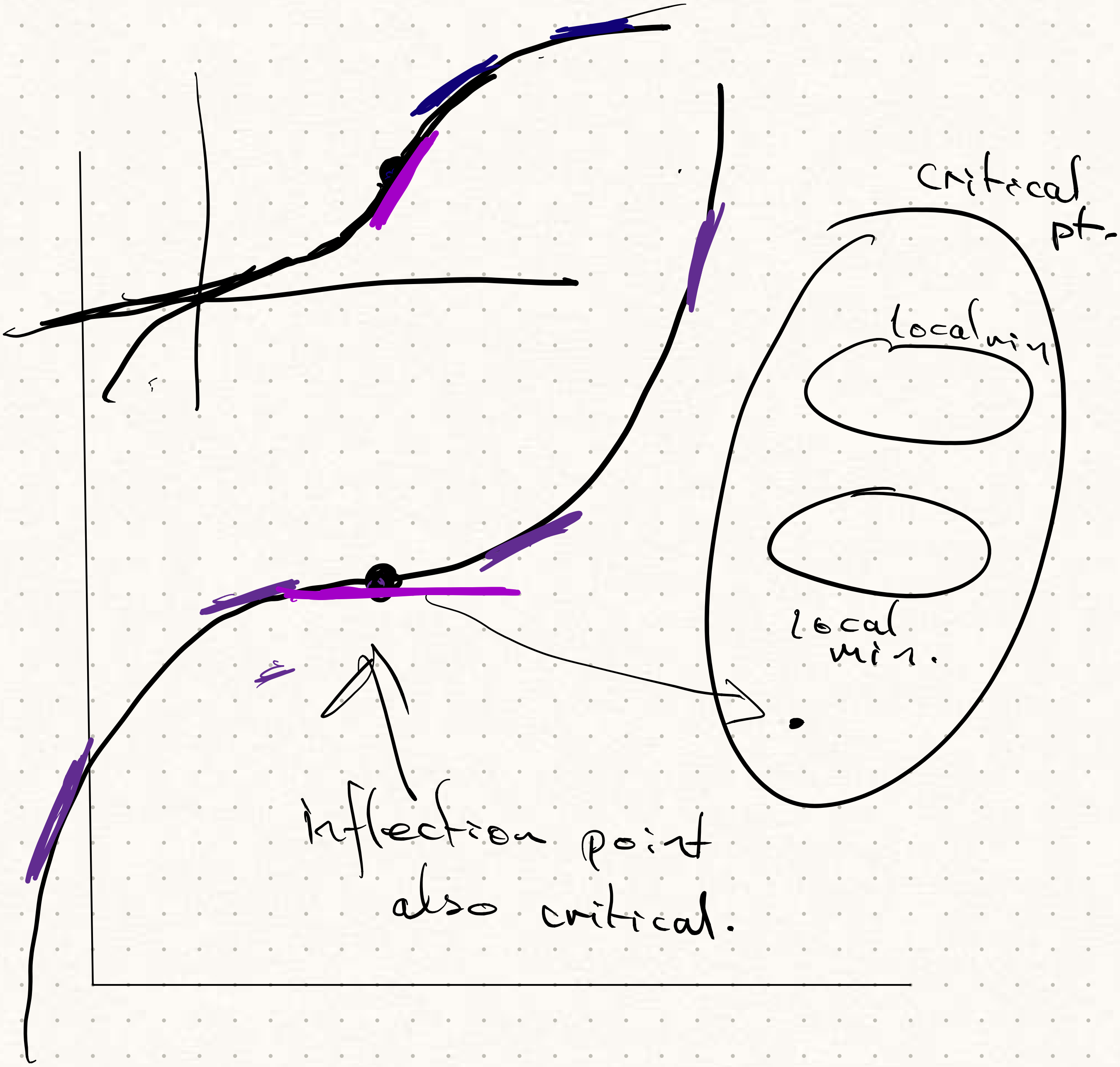
$-$ 0 $+$

f' ——— 4 ———



minimum

$f''(2) = 0.$



f increasing.

$\Leftrightarrow f' > 0$

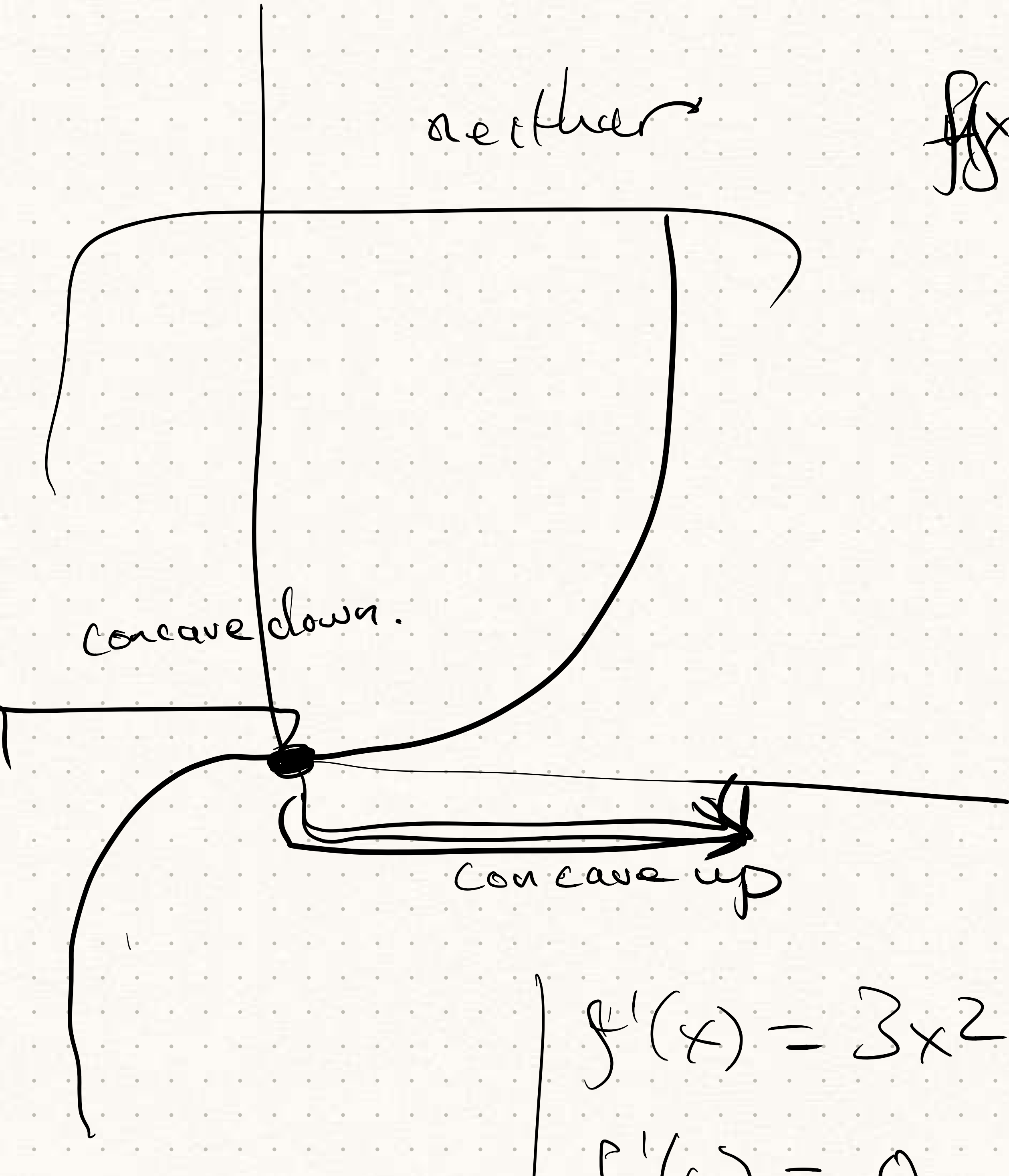
f' increasing

$\Leftrightarrow$ f concave up.

$f' > 0 \Leftrightarrow f$ increases

$f' < 0 \Leftrightarrow f$ decreases.

$f(x) = x^3$



$f'(x) = 3x^2$

$f'(\underline{0}) = 0.$

crit point

min

max