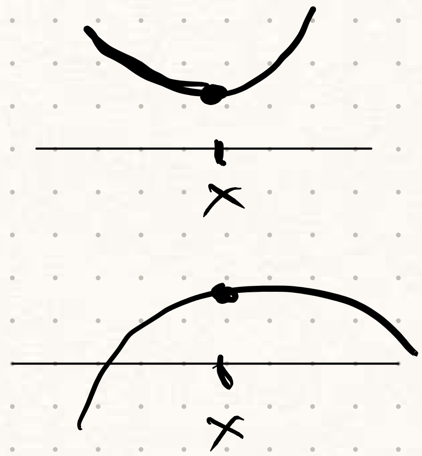
Lecture 29

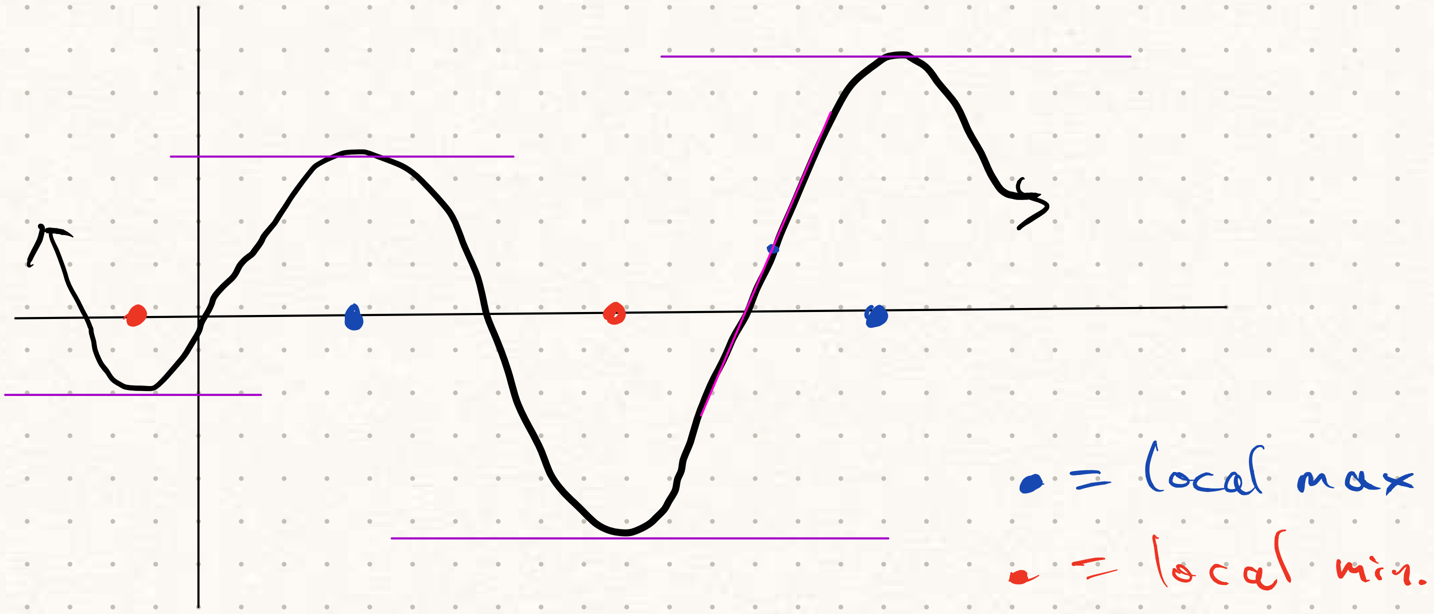
Last time:

Local minima: "valley"

Local maxima: "peak"

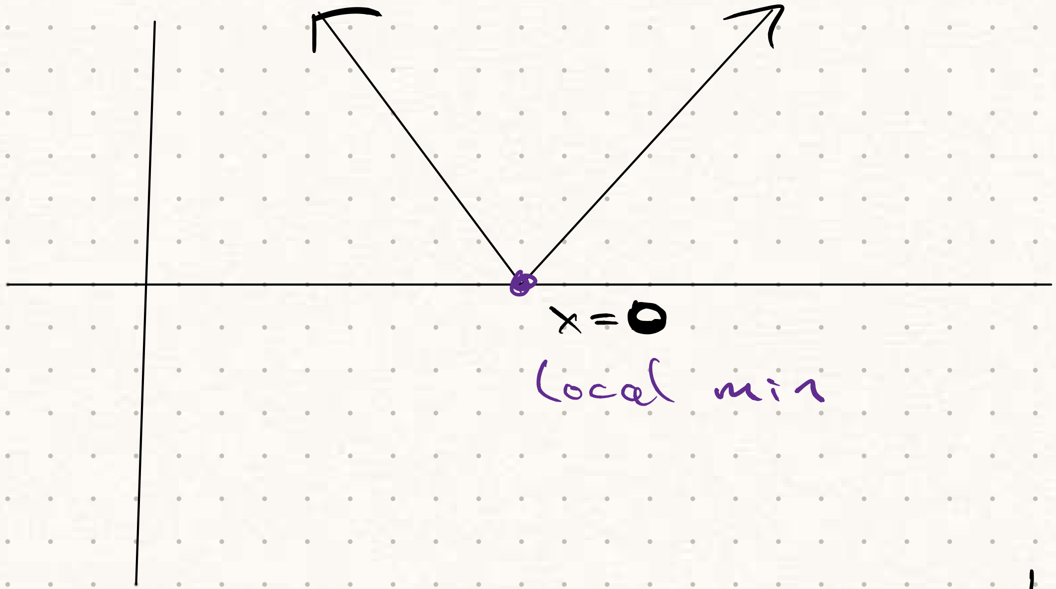


Example



Theorem: if f has local min/max at c, then c is a critical point.

Definition: c is crit point if $f'(c)=0$ or $f'(c)$ DNE.



$f(x) = |x|$

In this example, loc. min at $x=0$,

but $f'(0)$ DNE.

$$f'(0) = \lim_{h\to 0} \frac{|h|}{h}$$

DNE because

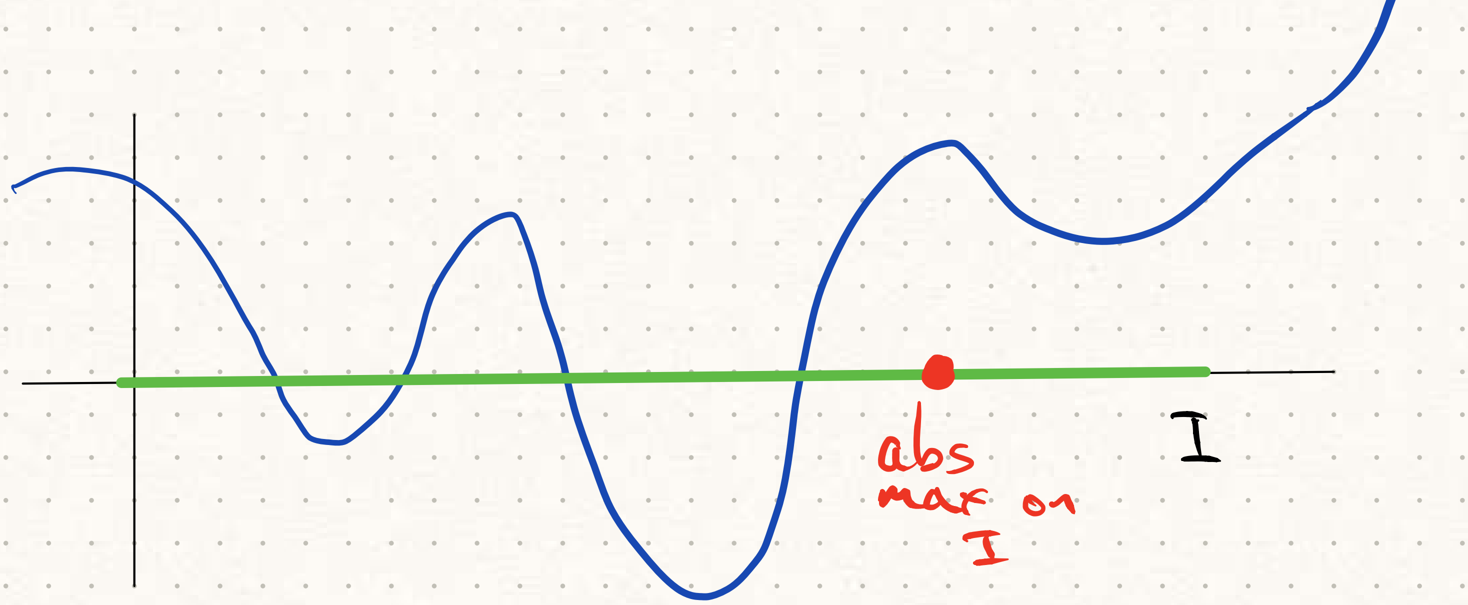
$$\lim_{h\to 0^+} \frac{|h|}{h} = 1$$

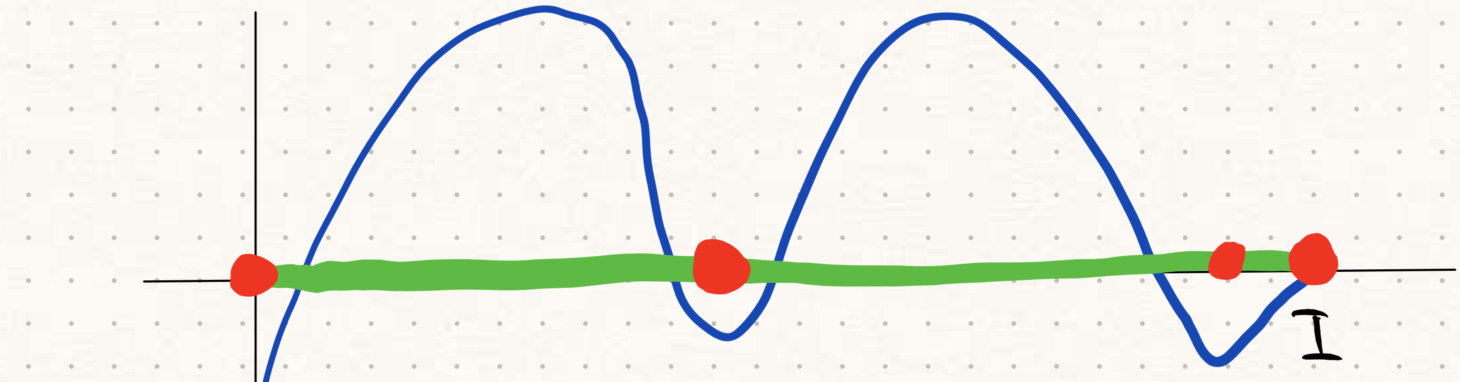
$$\lim_{h\to 0^+} \frac{|h|}{h} = -1.$$

<u>Definition:</u> local extrema $\Leftrightarrow$ local max/min.

Today Finding absolute extrema

Q: Given a function f, how to find the absolute min/max on interval I?





• = candidates for local min.

Key observation: abs min/max must be at

* one of the local mins

or

* one of the endpoints of I

We can use this to create a procedure for finding abs min/max:

Example: Find the absolute max and absolute min of

$$f(x) = -x^2 + 3x - 2 \quad \text{over } [1,3].$$

Soln:

I. Find the candidates:

(a) The critical points:

$$f'(x) = -2x + 3$$

$f'(x) = 0$ at $-2x + 3 = 0$

at $x = 3/2$.

1b) Endpoints: 1 and 3.

2. Evaluate f on the candidates

x	$f(x)$	Conclude
$\frac{3}{2}$	$\frac{1}{4}$	abs. max
1	0	
3	-2	abs min.

$$f(3/2) = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) - 2 = \frac{1}{4}$$

$$f(1) = -1^2 + 3(1) - 2 = 0.$$

Answer: abs min at $x = 3$

abs max at $x = 3/2$.

Ch 4.5 What f' tells us about shape of graph

Recall:

$f'(x) > 0$ means f is increasing ("uphill").

$f'(x) < 0$ means f is decreasing ("downhill").

Example

$$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x.$$

$$f'(x) = x^2 - 5x + 4$$

$$= (x-4)(x-1).$$

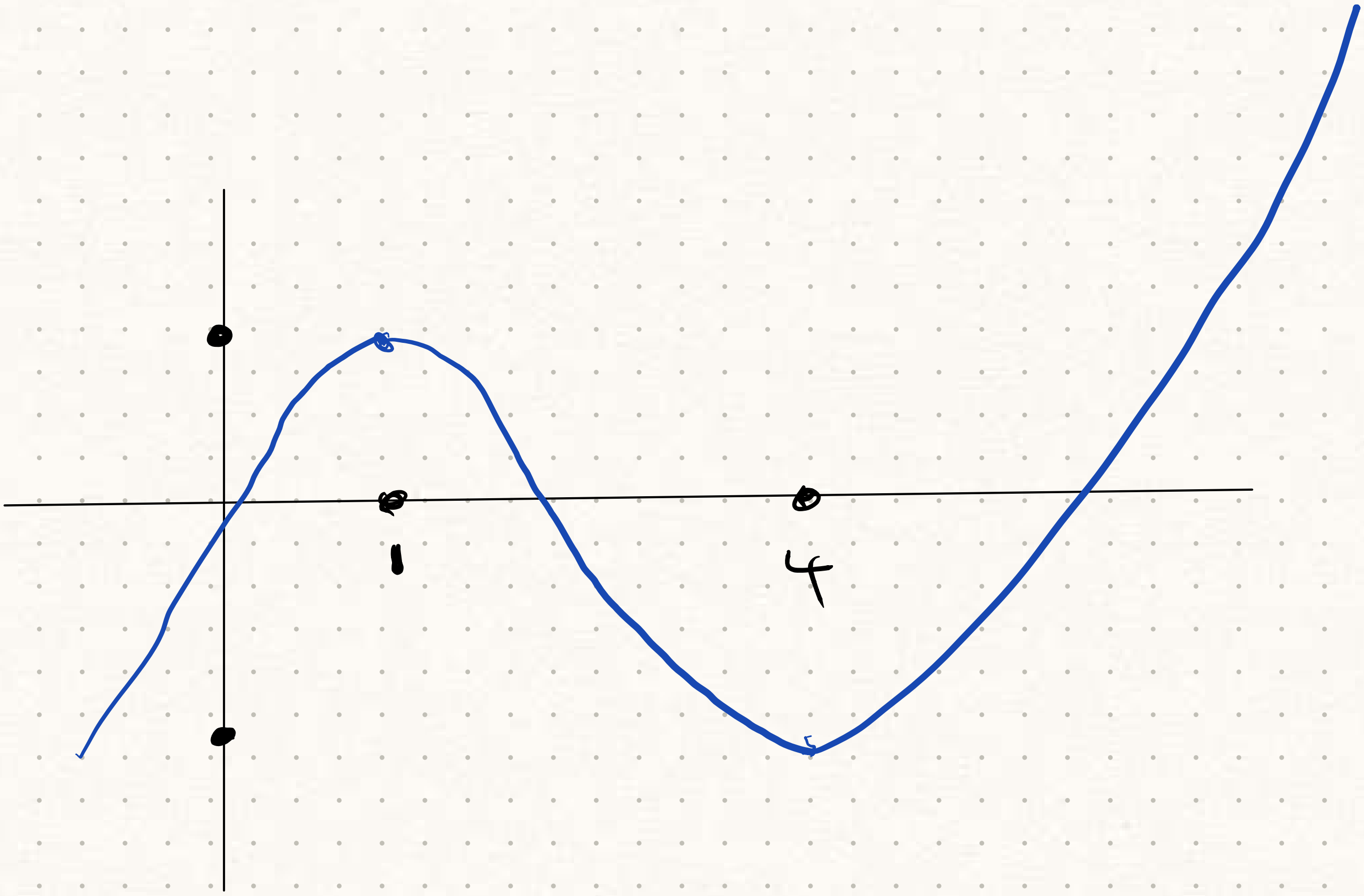
So

Since $x > 1$, $x - 1 > 0$
Since $x < 4$, $x - 4 < 0$.

Since $x > 4$
$x - 4 > 0$
$x - 1 > 0$

$f' > 0$ | 1 | $f' < 0$ | 4 | $f' > 0$

crit points.



$$f'(x) = \underbrace{(\overset{1.01}{x}-4)}_{\text{positive.}}\underbrace{(\overset{1.01}{x}-1)}_{\text{negative}}$$

If $x<4$ and $x>1$, then

$$f'(x)<0.$$