Plan for next few lectures

(4.3) (Maxima and Minima)

(4.5) (Derivatives and shape of graph).

(4.8) (L Hopital's rule).

Exam will be on:

3.3 - 4.3, 4.5, 4.8

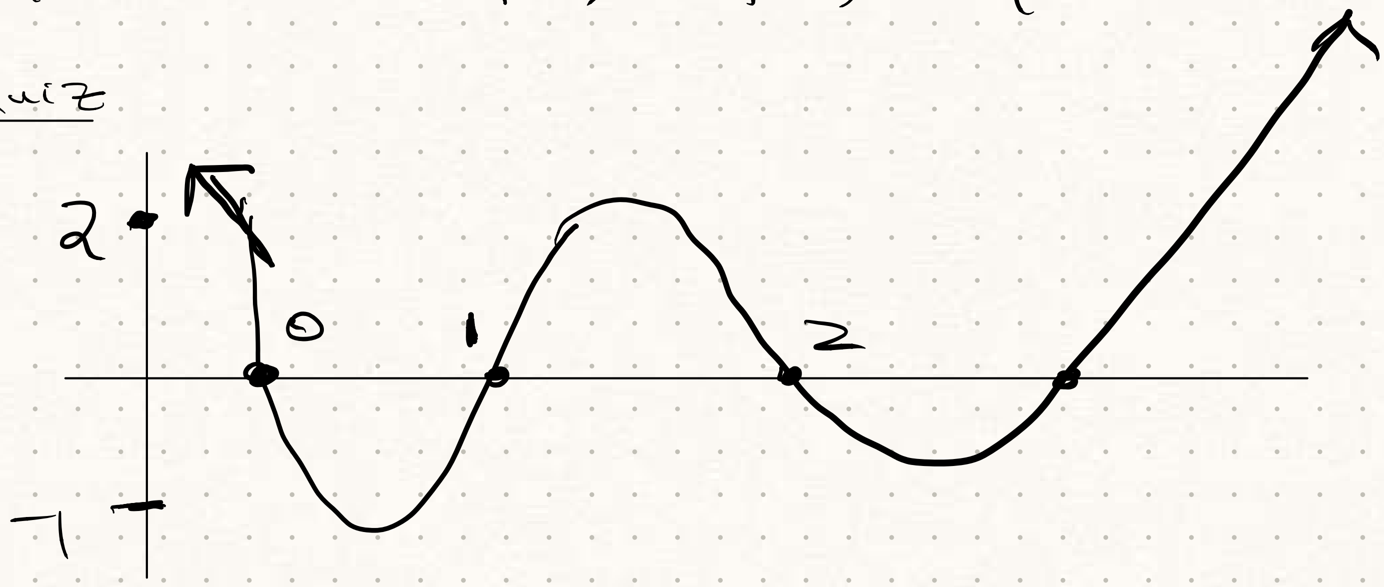
Practice exam out this week.

Last Lecture: We defined the sentence

"f has an absolute minimum at c".

It means $f(c) \leq f(x)$ for all x

Quiz



The abs. min of f is at

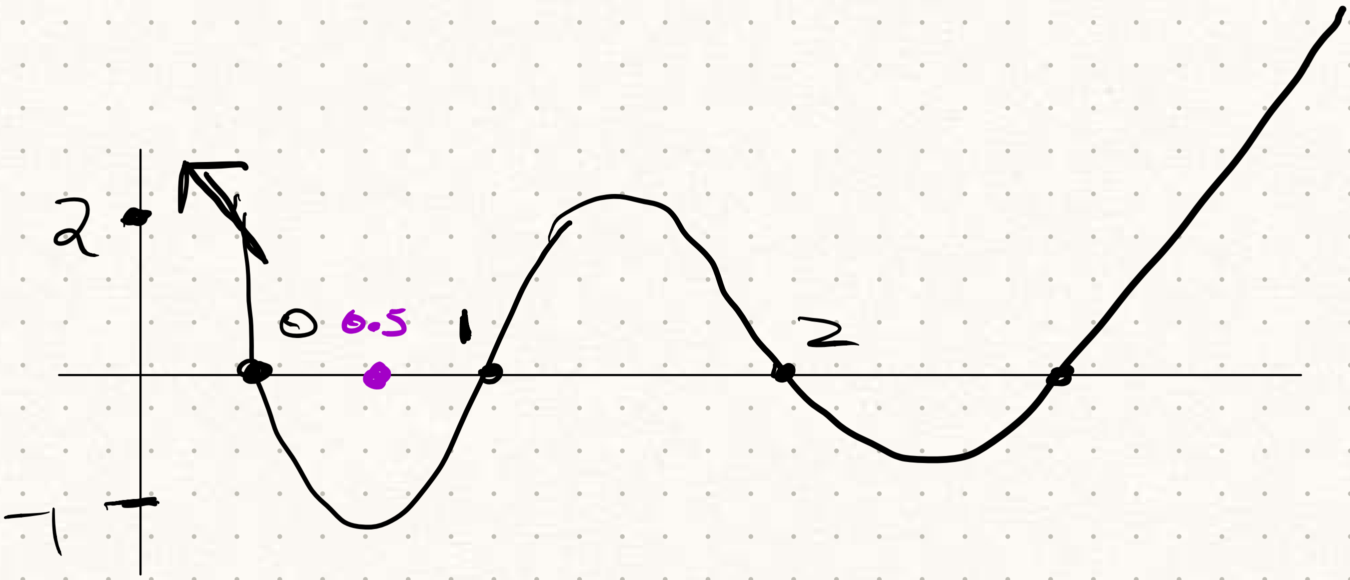
a) -1 ← 40%

b) -0.5

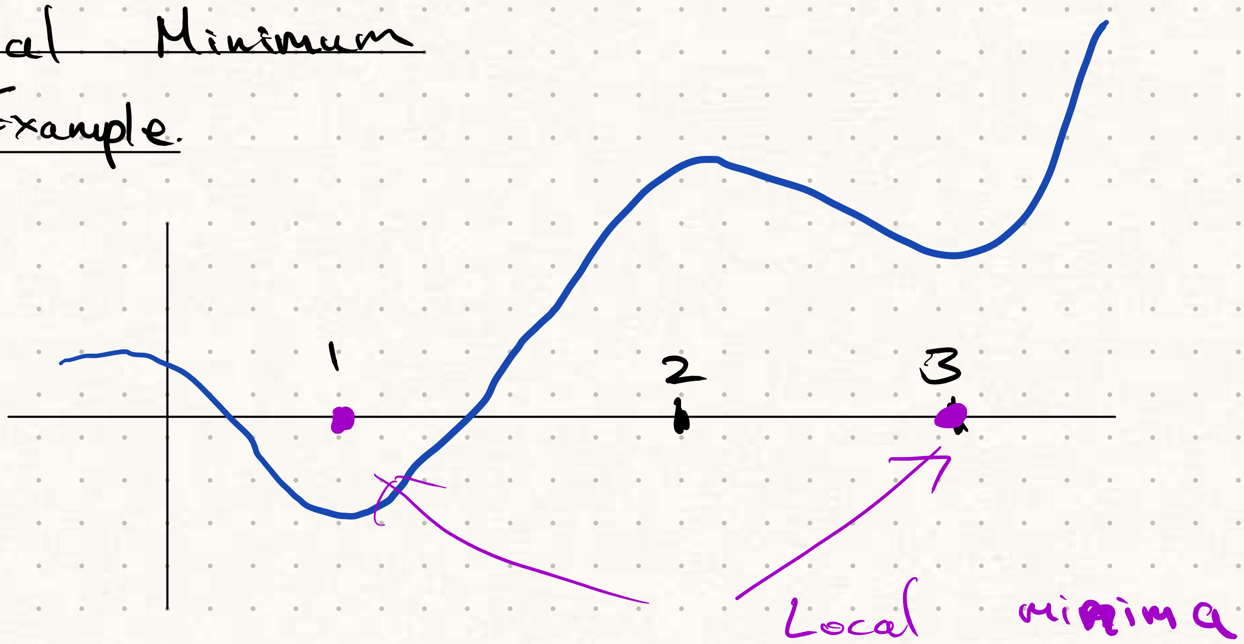
c) 0

d) 0.5 ← correct answer (35%)

e) 1



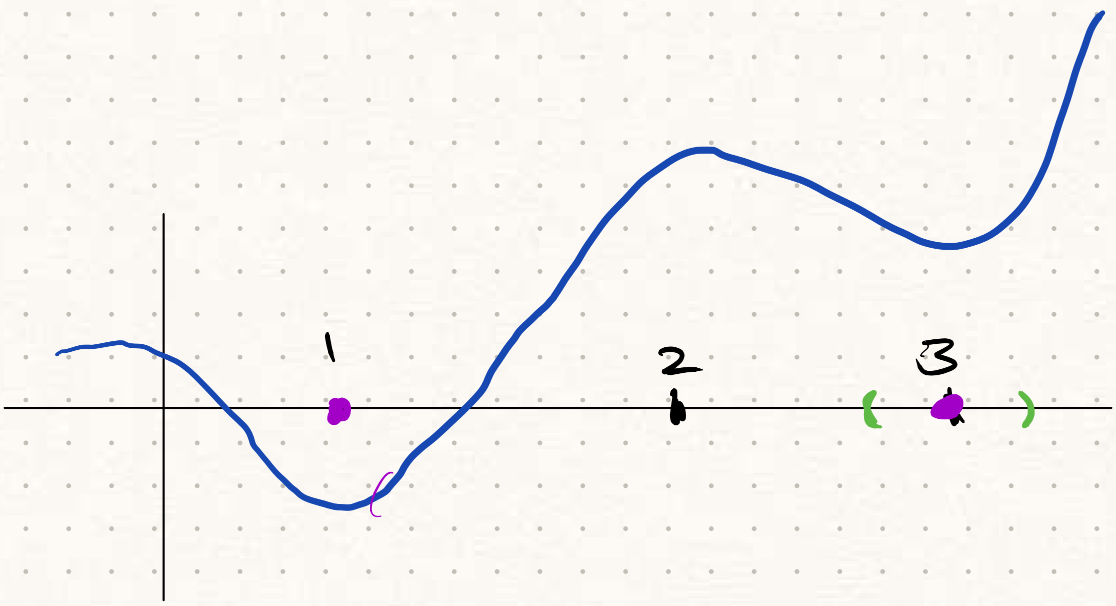
Local Minimum Example.

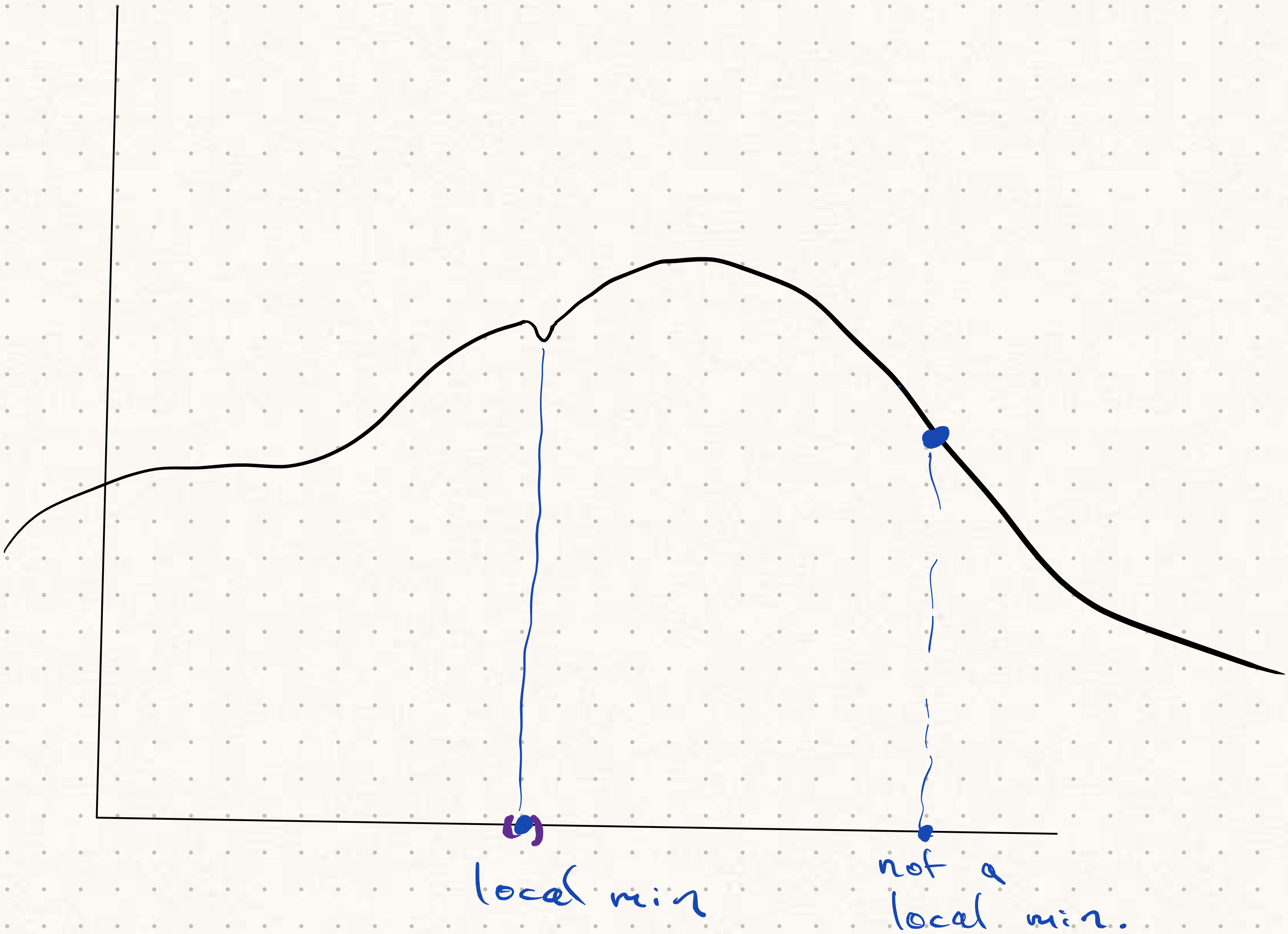


Definition f has a local minimum at c if

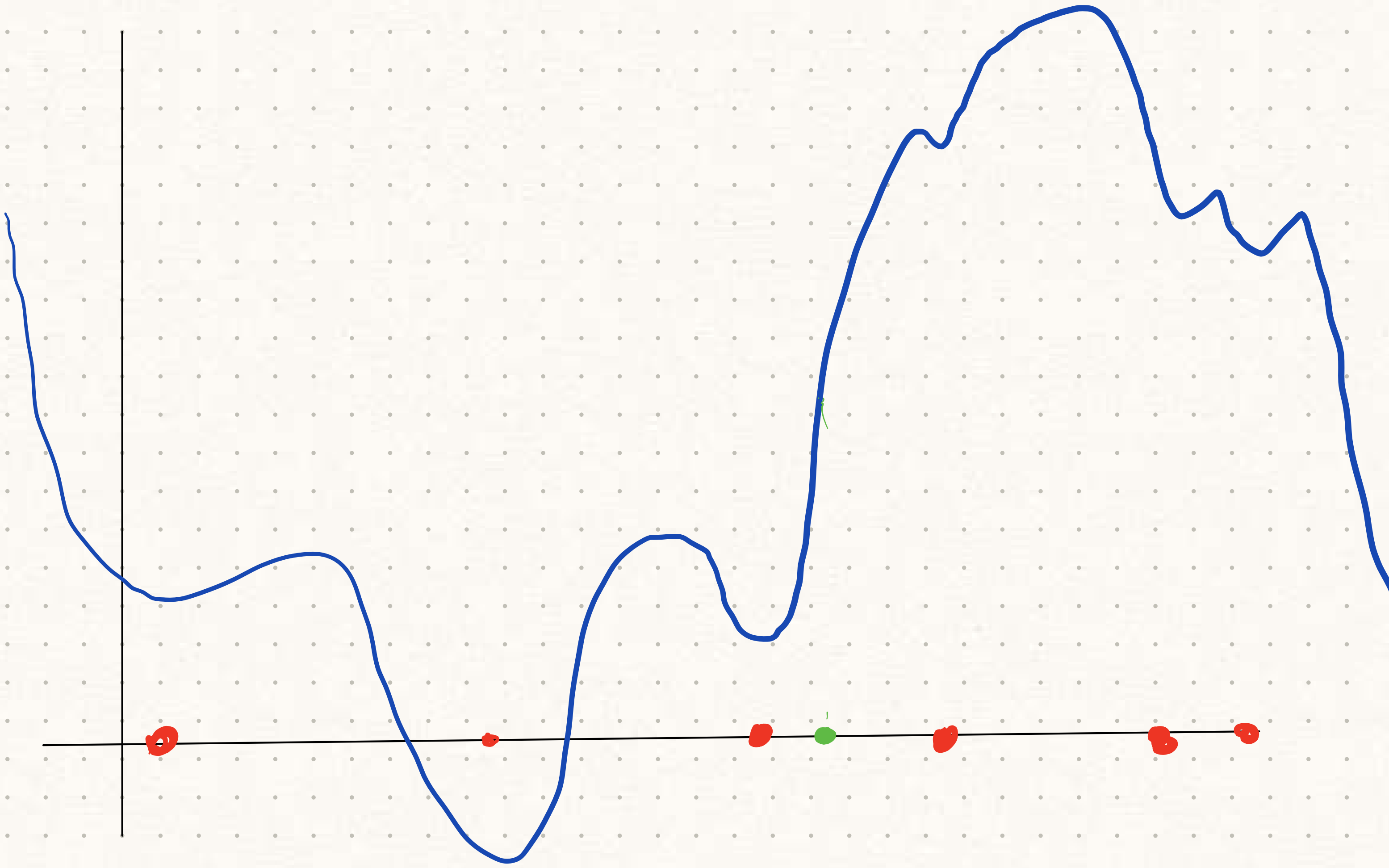
when you zoom in at $(c, f(c))$, it looks like an abs. min.

In other words, in a local neighborhood around c, c is a minimum.

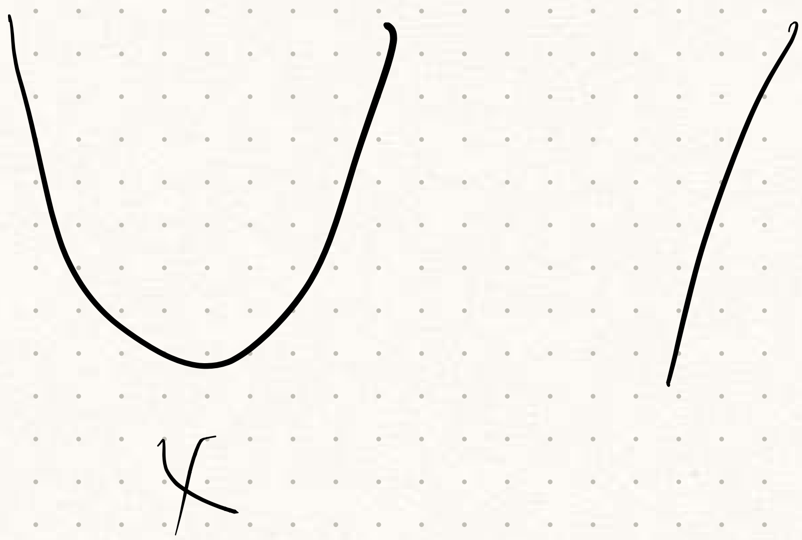




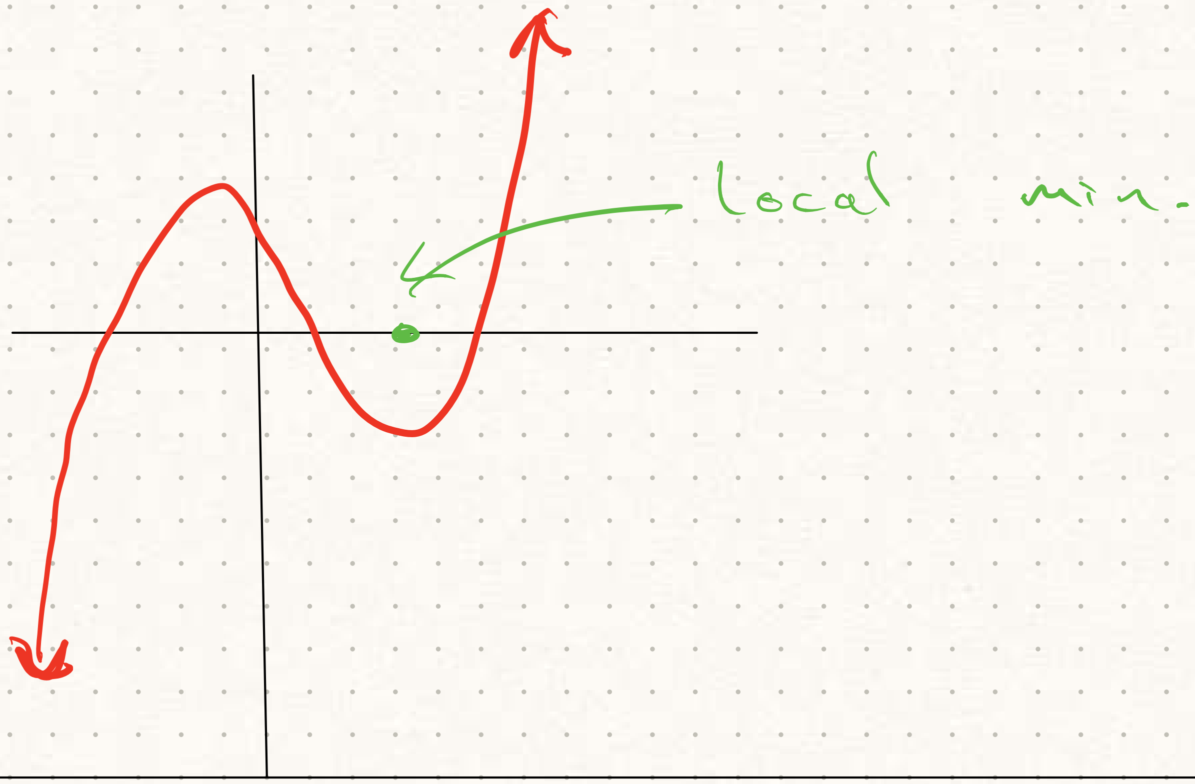
Similar definitions for local maximum and absolute maximum.



• = local minimum. (and also absolute minimum).



Some functions don't have absolute min.



Critical points

Definition:

c is a critical point for f if

$$f'(c) = 0.$$

Example

$f(x) = (x-1)^2 + 1$

$f'(x) = 2(x-1)$

So $f'(1) = 0$

So 1 is a critical point for f.

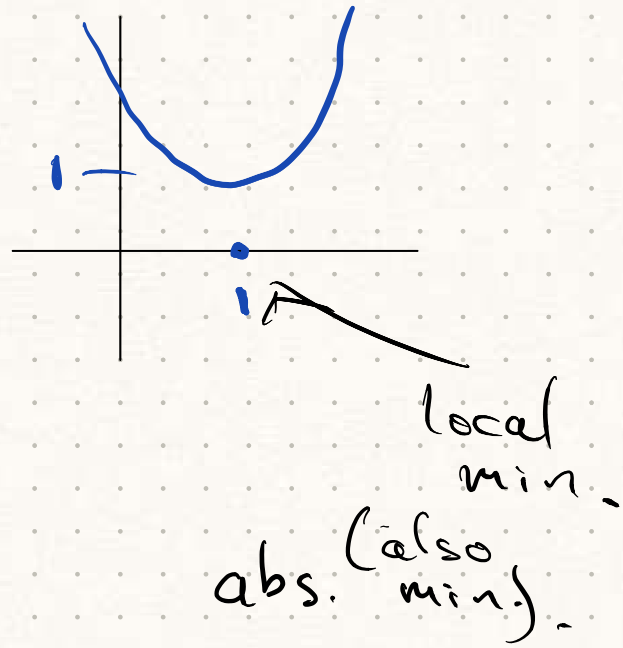
Theorem 4.2

if f has a local min. or local max, at c.

Then c is a critical pt for f.

Example

$f(x) = (x-1)^2 + 1$



* 1 is a local min

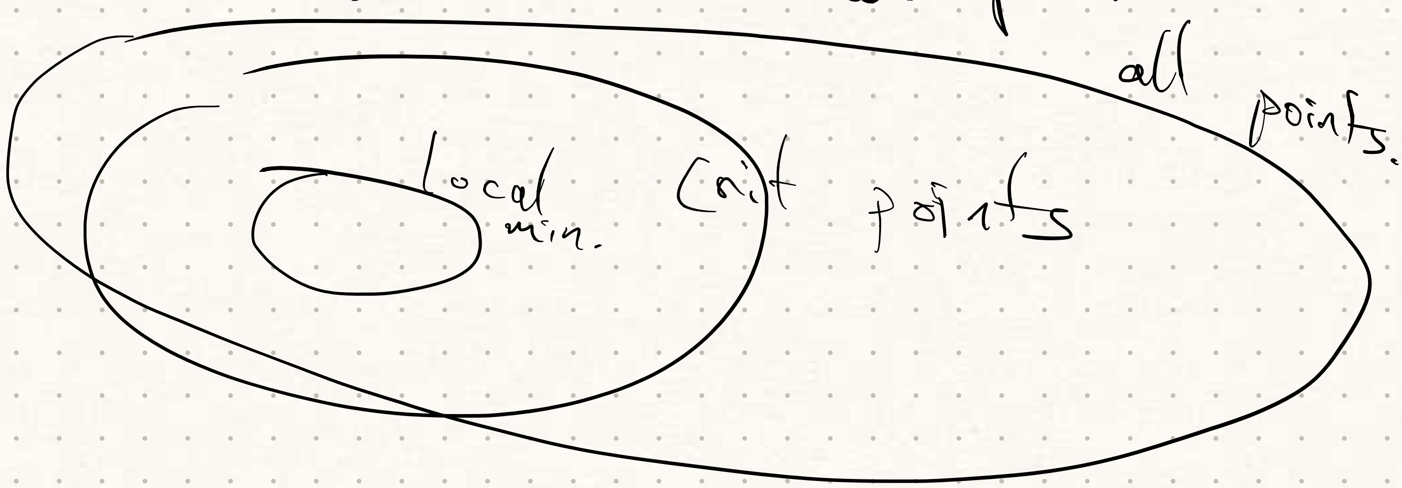
* $f'(x) = 2(x-1)$

  $f'(1) = 0$.

  f is a critical point.

Why this is useful:

if we want to the local min/max of a function, we only need to consider critical pts.



Examples

Example a)

$$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$$

Find critical points:

$$f'(x) = x^2 - 5x + 4$$

$$f'(x) = 0 \iff x^2 - 5x + 4x = 0$$

$$\iff (x-4)(x-1) = 0$$

$$\iff x = 4 \quad \text{or} \quad x = 1.$$

So crit. points are 4 and 1.

Theorem tells us that local min/max have to be at 4 or 1.

Verify via graph:

Why is this interesting:

* Finding the local minima/maxima feels like a "geometric" operation

* Finding critical points a very 'algebraic' operation.

It's interesting that these 2 operations are related.