Lecture 27

Last time:

estimate $f(x)$ near $x=a$

by its linear approximation

$$y = f'(a)(x-a) + f(a)$$

Example: $f(x) = x^2$, then

$$f'(x) = 2x, \quad f'(a) = 2a$$

so linear approx at $x=a$ is

$$y = 2a(x-a) + a^2$$

Choosing different values of a, gives different lines

See graph / animation.

Today: differentials (Ch. 4.2).

What we just did:

Used linear approx. to estimate function values

Another perspective: Using derivatives to estimate the change in a function.

Warmup Excercise

$$y = x^2 + 2x$$

"What is $\underbrace{\Delta y}_{\text{change in } y}$ if $x = 3$ and $\underbrace{\Delta x = 0.1}_{\text{change in } x}$".

" What is the change in y, if you change x from $x = 3$ to $x = 3 + 0.1$ "

Poll:

a) 15

b) 0.81 ← 50 %

c) 15.81 ← 25 %

d) 15.04

"What is the approximate Δy from $x=3$ to $x=3.1$"!

$(y = x^2 + 2x)$

Soln

$$\frac{dy}{dx} = 2x + 2$$

approx. change in x

differential

so $dy = (2x+2)\,dx$

so $dy = 8\,dx$ at $x=3$

so $\boxed{dy = 0.8}$ if $dx = 0.1$

Example 4.8 b)

$$y = e^x + \sin(x)$$

a) "What is Δy when $x=0$
$\Delta x = 0.2$"

b) Use differentials to estimate Δy.

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a) $\left(e^{0.2} + \sin(0.2)\right) - e^0 + \sin(0)$

$$\Delta y = 0.42007...$$

b) $\frac{dy}{dx} = e^x + \cos(x)$

$dy = [e^x + \cos(x)]\,dx$

$dy = 2\,dx$ at $x=0$

$dy = 0.4$ if $dx = 0.2$

Maxima and minima Ch 4.2

Quiz:

Where is the minimum of

$f(x) = x^2 + 1$

$x = 0.$

a) 0

b) 1

c) ∞

d) DNE

83%

Q: What is the minimum of

$f(x) = x^2 + 1$

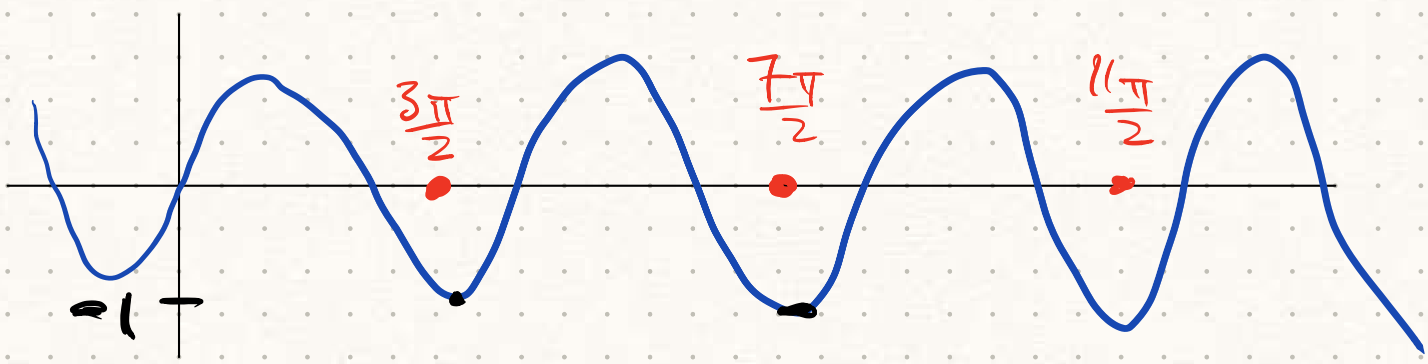
Ans: 1

Definition

An absolute minimum at 'c' means $f(c)$ is smaller (or equal to) than every other $f(x)$.

Example: Absolute minimum is at $x=0$ for $f(x)=x^2+1$.

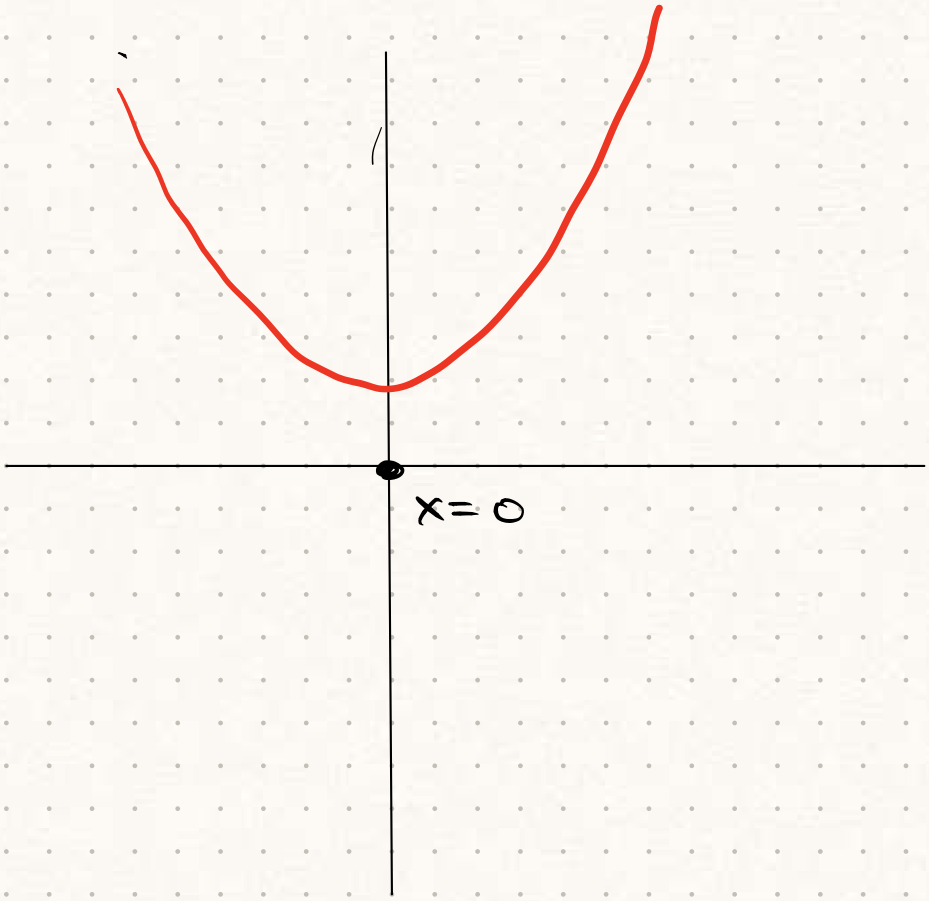
Example: absolute minimum of $f(x)=\sin(x)$ is at $\frac{3\pi}{2}+k2\pi$, $k\in\mathbb{Z}$



There are infinitely many locations for the absolute minimum.

Example

$$y = x^2 + 1$$



So abs min is at $x=0$.

Motivating question:

Minimum for

$$y = x^7 + \sin(x) + e^x - \tan(x) + \frac{x}{\log(x)}$$

See next lecture.

$$\mathbb{Z} = \{ -1, 0, 1, \ldots \}$$

$$k \in \mathbb{Z}$$

in

integers.