Lecture 25 Related Rates Ch 4.1

Key idea:

relationship between quantities $\Longrightarrow$ relationship between the rates of change of quantities.

In symbols:

Relationship between $V(t)$ and $r(t)$ $\Longrightarrow$ relationship between $\frac{dV}{dt}$ and $\frac{dr}{dt}$

one determines the other

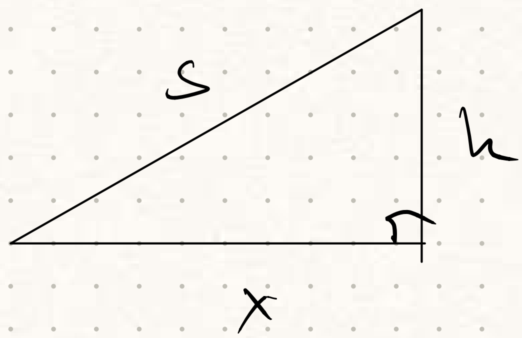
one determines the other

Last time we used

$$V(t) = \frac{4\pi r(t)^3}{3} \Longrightarrow \frac{dV}{dt} = 4\pi r(t)^2 \frac{dr}{dt}$$

Today: more complicated examples.

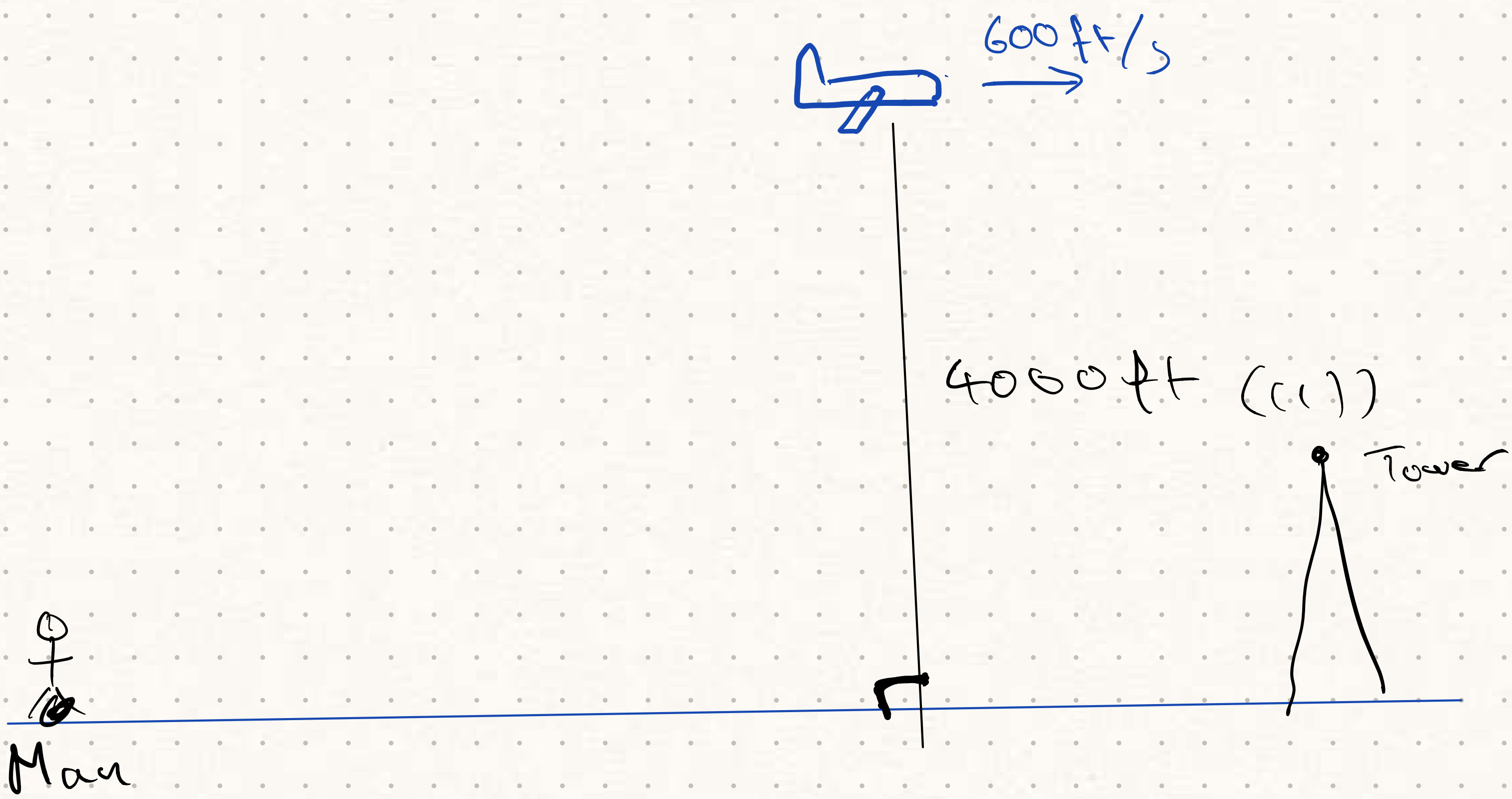
Review: Triangles



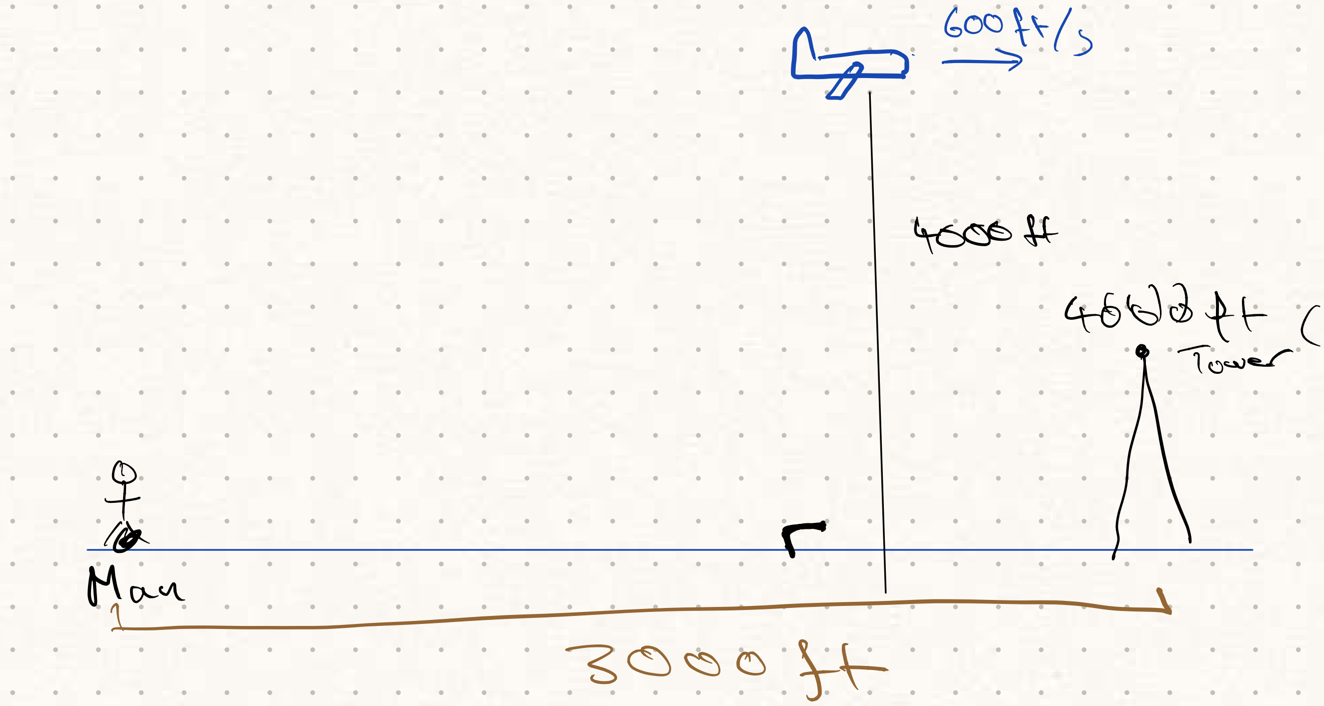
what is the relationship between x and s?

$$x^2+h^2=s^2 \qquad \text{(Pythagoras).}$$

<u>Problem</u> (Example 4.2 of textbook).



Problem (Example 4.2 of textbook).



* Plane at height 4000 ft

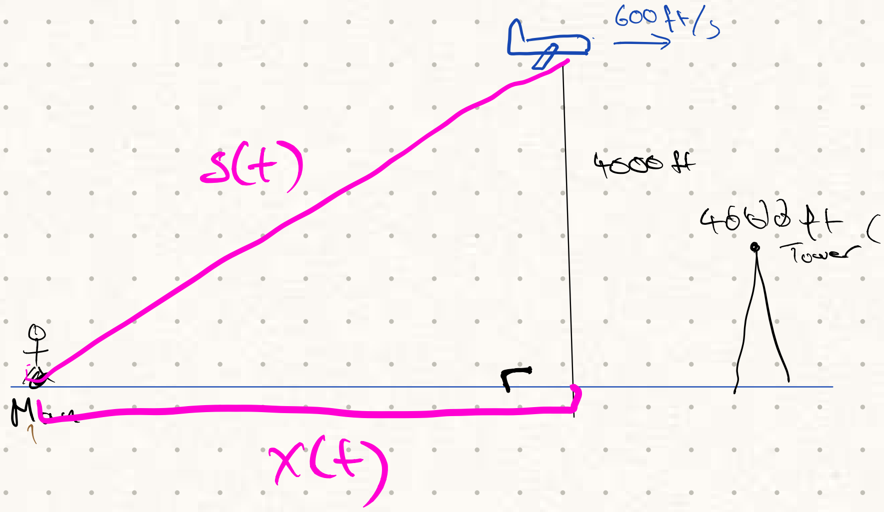
  Constant speed 600 ft/s

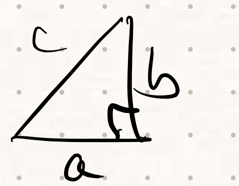
  from man to tower.

* Radio tower is 3000 ft from man.

Question: At what rate is the distance from man to plane changing, when plane passes over tower.

Step I: Draw a picture.
give names to the things that are changing.





$a^2+b^2=c^2$

Step 2. Write down info given, and what our goal is in terms of the variables introduced in step I.

$$\frac{dx}{dt} = 600$$

Goal: Find $\frac{ds}{dt}$ when $x(t) = 3000$

Step 3 Find a relationship between $x(t)$ and $s(t)$.

$$x(t)^2 + 4000^2 = s(t)^2$$

4) Differentiate both sides (apply $\frac{d}{dt}$ to both sides) to get relationship between $\frac{dx}{dt}$ and $\frac{ds}{dt}$

$$2\,x(t)\,\frac{dx}{dt} + 0 = 2\,s(t)\,\frac{ds}{dt}$$

5) Solve for the goal.

$$\frac{ds}{dt} = \frac{\cancel{2}\,x(t)\,\frac{dx}{dt}}{\cancel{2}\,s(t)}$$

$$= \frac{(3000)(600)}{s(t)}$$

Now $s(t)^2 = 3000^2 + 4000^2$

so $s(t) = 5000$

so finally $\frac{ds}{dt} = \frac{3000 \cdot 600}{5000}$

$= 360$ ft/s

$$s(t)^2 = 3000^2 + 4000^2$$

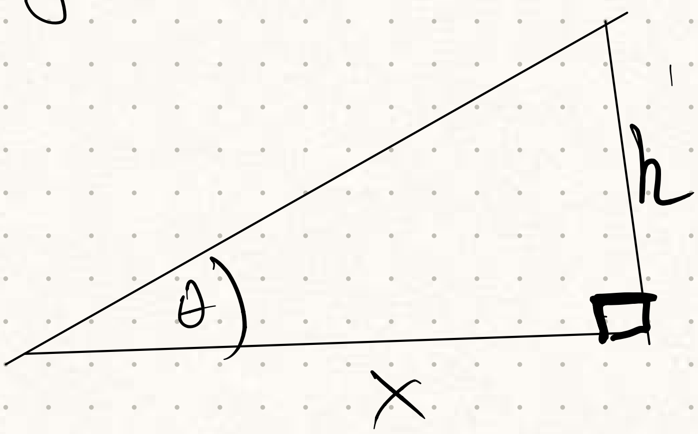
$$= 9 \times 10^6 + 16 \times 10^6$$

$$= 25 \times 10^6$$

$$s(t) = \sqrt{25 \times 10^6}$$

$$= 5 \times 10^3 . = 5000 .$$

Triangles Review

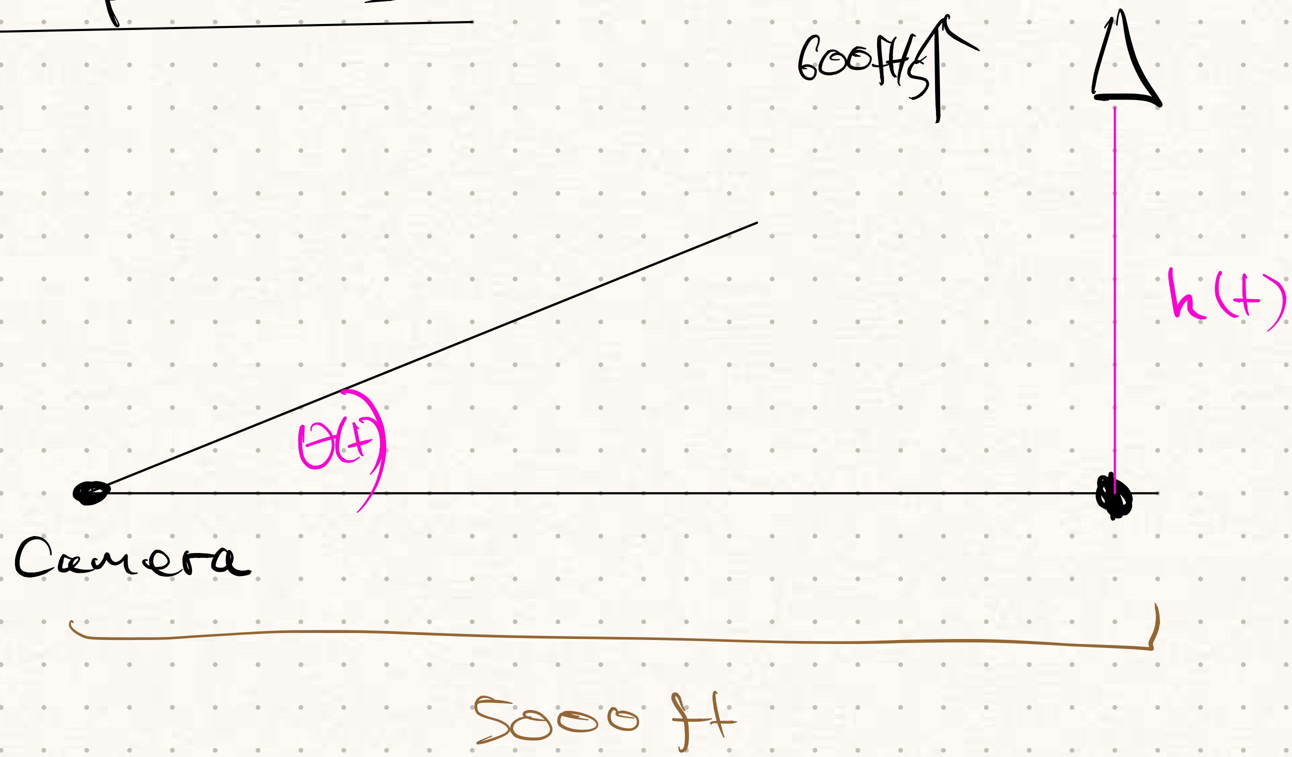


Relationship betwen θ and h?

$$\tan\theta = \frac{x}{h}$$

Rocket

Example 4.3



Question: What is the rate of change of the angle of camera, when rocket at 1000 ft.

Step 1 Draw the picture.
Label things that are changing.

Step 2

Given:

$$\frac{dh}{dt} = 600$$

Goal: $\frac{d\theta}{dt}$ when $h(t) = 1000$.

Step 3

Find rel. between h and θ.

$$\tan\theta = \frac{h(t)}{5000}$$

Finish problem by mimicing first example.

See textbook to get unstuck

Next time: Ch 4.2.

Question:

how do step 4 in first example.

Step 3:

$$x(t)^2 + 4000^2 = s(t)^2$$

Step 4) Apply $\frac{d}{dt}$ to both sides

$$2x(t)\frac{dx}{dt} + 0 = 2s(t)\frac{ds}{dt}.$$

Why is $\frac{d}{dt}x(t)^2 = 2x(t)\frac{dx}{dt}$?

Chain rule: $\frac{d}{dx}x^2 = 2x$

$$\frac{d}{dx}\sin(x)^2 = 2\sin(x)\cos(x)$$

$$\frac{d}{dx}f(x)^2 = 2f(x)\frac{df}{dx}$$

$$\frac{d}{dt}x(t)^2 = 2x(t)\frac{dx}{dt}$$

$f \to x$

$x \to t$