Homework 10

Due Monday April 9th at the beginning of class.

1. Prove that

$$\frac{\pi}{2} \;=\; \lim_{n\to\infty} \frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdot\dots\cdot(2n)\cdot(2n)}{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot\dots\cdot(2n-1)\cdot(2n-1)}$$

(using sine product series).

2. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function. Find necessary and sufficient conditions so that

$$f(z) \;=\; e^{p(z)} \qquad \text{where } \; p(z) = \sum_{k=0}^{N} A_k z^k$$

is a polynomial. (Answer should involve determinants.)

3. Prove that

$$\sqrt{\pi}\,\Gamma(2z) \;=\; 2^{2z-1}\Gamma(z)\Gamma(z+\frac{1}{2})$$

using anything proved in class.

4. Show that

$$\prod_{\text{primes } p}\left(1-\frac{1}{p^z}\right) \;=\; \sum_{n=1}^{\infty}\frac{\mu(n)}{n^z}$$

where

$$\mu(n) \;=\; \begin{cases} 0 & \text{if } n \text{ has a square divisor} \\ (-1)^k & \text{if } n = p_1\cdots p_k. \end{cases}$$

Where does the product converge? Where does the sum converge?

5. From your text:

No. 5 on page 198.