# Selected reviews of NSF proposal

Included below are a selection of reviews of my research proposals to NSF in 2000, 2003 and 2006. I have chosen a subset which avoid too many technical details and restricted to the 'intellectual merit' portion of the reivew. The proposals and the full text of all reviews are posted at http://www.math.sunysb.edu/~bishop/vita.

#### 2000, Review 1

I rate this proposal as EXCELLENT, very close to the top.

The proposer raises 25 problems/conjectures, each of which making connections with conformal mappings, probability, Kleinian groups, quasiconformal analysis, in all ranges and permutations. It is a splendid list.

The proposer has made powerful impressions in several areas in geometric function theory, able to do hard estimates with a broad vision of what seems significant in the broad sense, and is able to connect the strands together. He has an active program whose value shows in that students of top analysts migrate to Stony Brook to learn and interact with him.

The effect is reciprocal; for example conjecture 15 was settled in two dimensions with another post-doc, and now proposer offers a bold way of bringing this to higher dimensions. This is related to a topic one of his graduate students is pursuing, according to the proposal.

Brennan's so-called conjecture on the integrability of the derivative of a conformal mapping has been around for probably 20 years, but it has provided a vehicle for introducing several exciting new approaches (all of which have, in the end, failed to resolve the original problem!). The proposal has a new angle: his decomposing (which is to appear, and is not accessible from his web page, so I could not consult it first-hand) of any conformal map into K-qc homeomorphic and an 'expanding' conformal factors. This leads to Question 1 which, even if not proved, already shows significant connections with outstanding work of Astala, Makarov, and the Sullivan convex hull theorem (which is where his weak bound for Question 1 arises). This leads to many related and interesting questions, passing though Conjecture 9.

A second set of problems concerns the Ahlfors conjecture on the area of the limit set of a finitely-generated Kleinian group. It appears that the proposer has been working on this subject for several years, but the joint paper (Acta 1997) with Peter Jones on the relation between limit set and exponent of convergence is an enormous step, and in turn leads to a path connecting this problem to a dynamic one involving the decay of heat kernels

The thoroughness and scope of the proposal make it one of the five-ten best I have review in the last decade. There is a cornucopia of ideas, even a half-page devoted to problems that would be suitable for undergraduate research projects. It is a virtuoso display.

### 2000, Review 2

This is a superb proposal of a highly productive mathematician with a full tool kit allowing him to roam with seeming ease and insight over complex analysis, geometry and probability. Much of Bishop's past work is brilliant, in particular his work with Peter Jones and his success in bringing the study of the hyperbolic convex hull out of the closet. Among many others, his ideas on bringing the study of the heat kernal in to study limit sets are exciting—Ahlfors would have appreciated this very much. Who knows what he will discover next? Bishop has enough ideas to keep a veritable army at work. His presence at a conference automatically increases intellectual excitement and interaction between participants. Anyone who thinks complex analysis is dead should read this wide ranging and highly stimulating proposal.

#### 2000, Review 6:

The proposal includes a number of original ideas. I don't know all the areas that this proposal touches on, which are quite numerous, reflecting Bishop's energy and breadth. With respect to Brennan's Conjecture, he has opened up an entirely new approach, which may well be successful. In any case, attempts along the lines suggested will open up new avenues for research.

Bishop's proposal points to interesting connections between hyperbolic geometry, differential geometry, global analysis and the theory of functions of one complex variable. Working at Stonybrook with Yair Minsky around, Bishop has a particularly good chance to make significant contributions to the study of Kleinian groups, where he knows considerably more analysis than most people in the field.

Although I have never met him, I understand that he is a very stimulating person to talk to, with ideas in many subjects. His work has had a considerable influence on my own recent research.

## 2004 Panel Summary

This is an excellent proposal. The PI is one of the most original and imaginative researchers in his area. The problems proposed, on conformal collapsing maps, Kleinian groups and connections of 3-dimensional geometry to numerical analysis of conformal mappings, are very interesting and innovative, some of them extremely difficult. He has an excellent record. His work covers a wide area, using very different and new ideas.

#### 2003, Review 3

Bishop is one of the most innovative researchers in geometric complex analysis and related topics. Has an impressive record of research, and the proposal gives strong indications of this continuing in the future. His most recent work covers a exceptionally wide spectrum of topics: conformal welding, interpolation of conformal mappings, introduction of methods of three dimensional hyperbolic geometry to conformal mappings, applications to numerical analysis, several interesting works on Kleinian groups, Rudin's orthogonal conjecture, conformal dimension and properties of quasiconformal mappings.

The proposal has three different research programs, each of them important and promising. The first on conformal collapsing suggest a unification and an approach to several questions in geometric analysis in one complex dimension. The unifying aspect comes through understanding the conformality properties of Moore's theorem, and I found this part of the proposal especially innovative. Many of the problems suggested here are notoriously difficult, but even success with partial results would have an impact. The second theme considers Kleinian groups and is continuation of Bishop's earlier work. Several central conjectures such as Ahlfors' conjecture are listed here, but it is not clear if there are new methods to approach this set of problems. On the other hand, the last theme, on applications of ideas from the three dimensional topology to the numerical analysis of conformal mappings and Schwarz-Christoffel formula I find again very innovative and promising for a major breakthrough in the near future.

## 2006, Panel Summary

The PI is a clear leader in his field with an excellent track of solving difficult problems and of a recognized high technical ability. In this proposal he lists a large number of interesting and important problems on conformal and quasiconformal mappings, many of them of a computational nature. The panel felt that this is important work that should be supported and that the PI has the ability to make significant progress on hard problems.

# 2006, Review 4

In this proposal Chris Bishop poses a large number of problems on conformal and quasiconformal mappings of plane domains. In the first half of the proposal, the problems arise out of how to compute the map from a Euclidean polygon onto a disk. He already has an algorithm to compute a  $1 + \epsilon$  quasiconformal map from an n-gon whose complexity is  $O(np \log p)$  where  $p = |\log \epsilon|$ . He now asks is current algorithm optimal - can the  $O(p \log p)$  term be improved. Can he use a "faster" FFT (fast fourier transform) algorithm since he only needs an approximation of the FFT. What about a faster method for Schwarz-Christoffel mapping? He has a conjecture on bit complexity. Can he extend the algorithm to circular arc polygons. What about Koebe domains? And this is only the beginning.

These problems seem to be his current interest and he has three preprints in the last year on them.

In the second part of the proposal, Bishop asks a number of questions about chordarc curves: among others, is the space of them connected in the BMO topology? can a chord-arc curve be straightened by an expansive motion? He also has a number of problems about conformal welding. These problems are more in the line of what Bishop has done for most of his 20 year career. He has a strong track record of finding good counterexamples to conformal (and quasiconformal mapping problems). These, in turn, often show what the theorems should be. He has terrific technical ability and a broad outlook.

### 2006, Review 6

The general area of this proposal is classical complex analysis very broadly construed. Specific problems in the proposal deal with efficient algorithms in computational conformal geometry, the geometry of chord-arc curves in the plane, distortion of quasiconformal maps, and conformal welding.

The PI is a leader in the field and has an outstanding record of contributions to the area with many publications in top journals. The PI's choice of problems shows a broad vision of the field. Many of his ideas are very innovative and may lead to a solution of Koebe's longstanding problem on uniformization by circle domains or to a solution of the important problem of characterizing the plane up to bi-Lipschitz homeomorphism, for example. Given the PI's past record, it is guaranteed that the proposed activity will lead to top-quality research with high impact.