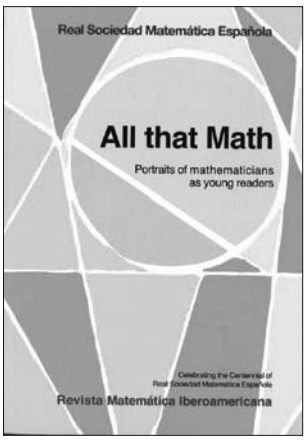

Antonio Córdoba,
José L. Fernández,
Pablo Fernández (editors)

All that Math
Portraits of mathematicians as young readers

Revista Matemática Iberoamericana, Madrid, 2011.
Paperback, x+346 pages
ISBN: 978-84-615-2900-1.

Reviewer: Vagn Lundsgaard Hansen

Most people remember their first day in school, their first love and other important moments in their lives. Like other scientists, research mathematicians also remember when they first experienced the very special excitement of making a (mathematical) discovery leading to publishing their first scientific paper. Quite often they also clearly remember the events that brought them on the track to their discoveries. In many cases it was reading a paper, in other cases it was attending a lecture and very often it was stimulated by mathematical discussions with fellow mathematicians. We all have our small stories to tell about how we got our first original idea and what happened in our attempts to develop it. But are there common features in all these personal stories? I personally think so, and I find that the book under review supports this view in an excellent manner by presenting the recollections of their entrance into research by 34 distinguished mathematicians from all over the world.

In 2011, the *Real Sociedad Matemática Española* asked the editors of the journal *Revista Matemática Iberoamericana* (founded by the society) to devote a special isue of the journal to celebrate the society's centennial. The editors are to be commended for the idea of asking a group of authors and members of past and present editorial boards of *Revista* to contribute to this special issue "with an essay about a paper – not necessarily the most important publication in the field – which, in one way or another, had a deep impact on their own mathematical careers, especially at its early stages".

The essays are of a very diverse nature. Some essays are rather demanding from a technical point of view while others are more discursive with emphasis on personal reminiscences about eminent mathematicians from recent times. Of the more technical essays, I personally liked "A random walk in analysis" by the American mathematician Christopher J. Bishop, reporting on important developments in conformal geometry. Of the biographically oriented essays, I found great pleasure in reading "Mathematical encounters" by the Czechoslovakian born mathematician Joseph J. Kohn, which contains interesting and amusing information about great mathematicians such as Norbert Wiener, John F. Nash, Solomon Lefschetz and, not least, Kohn's thesis advisor Donald C. Spencer. In a very short essay "Some recollections", the American mathematician Stephen Wainger declares his great debt for becoming a mathematician to several books by Eric Temple Bell, in particular his famous book *Men of Mathematics*, thereby demonstrating that good popularization can be of importance in stimulating young people to a career in mathematics.

The group of authors also includes the American Fields Medallist Charles Fefferman, who in the essay "A reminiscence on BMO" describes his reading of a paper by Eli Stein on singular integrals and differentiability properties of functions while a graduate student at Princeton University. Arriving in 1970 at the University of Chicago as a new assistant professor, Fefferman was challenged by a question from Antoni Zygmund to find a characterization of functions in BMO in terms of the Poisson integral. Fefferman realised quickly how he could use techniques acquired by reading the paper by Stein to answer Zygmund's question and solved in fact a more general problem in less than two weeks. Fefferman's final remark about this experience deserves quotation: "For many years, I've worked very hard to prove theorems. With luck, I've found complicated proofs after much suffering. With extraordinary luck, I found simple proofs after even more suffering. To find a simple proof, without suffering for it, is a very rare success. I will always be grateful for my incredible luck in reading Eli's paper and hearing Zygmund's question."

The stimulation you can obtain by attending a general lecture by a real master is vividly recorded in the essay "Olé!" by the Spanish mathematician José L. Fernández. The essay takes the point of departure in a lecture by Mark Kac that the author attended in 1984. It takes us on a tour through random walks, martingales, geometric function theory and, finally, conformal mappings. In the latter area the author singles out a fundamental paper close to his heart published by Nicolai G. Makarov in 1985 in the Proceedings of the London Mathematical Society. The essay ends with a statement about beauty in mathematics: "Appreciation of beauty is a cumulative cultural affair continuously evolving: you have to be prepared and ready to discriminate, to appreciate and to *share* it. But to *create* beauty, well, that is altogether a different matter. *Olé Makarov*!"

During the wonderful "6th European Congress of Mathematicians" in magnificent Kraków, Poland, at the beginning of July 2012, I had many opportunities to think about the proud traditions of Polish mathematics, which become even more impressive taking into account the many dark moments in Polish history. I was therefore happy to find in the book an essay "Let the beauty of Harmonic Analysis be revealed through nonlinear PDEs" by the Polish mathematician Tadeusz Iwaniec. He is obviously a highly spirited person and his essay contains a lot of amusing comments and quotations, among others a self-ironic one by the New Zealand mathematician Gavin Martin, another contributor to the book, who says about Iwaniec: "After all, he has quite a good memory even if it is a bit short." But there is also a short, gloomy paragraph in the essay where Iwaniec says: "As I

share the beauty and joy of mathematics with you I also remember Polish mathematicians whose glorious scientific careers came to a cruel end during Nazi-Soviet occupation. *Józef Marcinkiewicz*, *Stanisław Saks* and *Juliusz Paweł Schauder* were inspirations to me."

Among the many exciting essays in the book, I shall here only mention one more, namely the inspiring essay "Two papers by Alberto P. Calderón" written by the French mathematician Yves Meyer. In the essay, Meyer describes two short papers by Calderón, which were pivotal not only to his own research work but which – in his opinion – completely changed the paths of real analysis, complex analysis and operator theory for the future of mathematics by their elegance, conciseness, profoundness and vision.

The essays in this book describe some of the first experiences with contemporary research that a young mathematician meets when entering a research career in mathematics. The topics cover a wide scope of branches of mathematics and the exposition in many of the essays will therefore, in places, inevitably challenge any reader, especially since the authors of the essays attempt to give a fair description of the scope and depth of the mathematics behind the essays. Nevertheless, most of the essays are in my opinion valuable contributions to understanding the way research mathematicians think and work, in particular, how they get their ideas. I congratulate the *Real Sociedad Matemática Española* on their first hundred years and especially on the publication of this inspiring and valuable commemorative issue of their journal *Revista Matemática Iberoamericana.*

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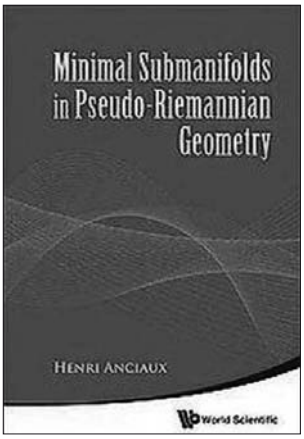

Henri Anciaux

Minimal Submanifolds in Pseudo-Riemannian Geometry

World Scientific Publishing Co., November 2010
184 pages
ISBN: 978-981-4291-24-8

Reviewer: Magdalena Rodríguez

The classical theory of minimal surfaces in Euclidean space, whose roots go back to the beginning of the calculus of variations in the 18th century, while still enjoying a great amount of research effort nowadays, has motivated the emergence of the theory of minimal submanifolds in Riemannian manifolds, an extremely active field of research of its own. And it is raising an interest for generalising such a theory to a wider range of ambient spaces. Minimal submanifold theory involves techniques from different areas, including partial differential equations, complex analysis, algebraic geometry and geometric measure theory. On the other hand, the study of pseudo-Riemannian manifolds was developed in the last century, in part due to its importance as the main tool of the theory of relativity. In this book the author combines an interest in both topics, studying minimal submanifolds in pseudo-Riemannian manifolds (the concept of minimal submanifolds must be understood in a wider sense, which includes also, for instance, maximal hypersurfaces in Minkowski space, largely studied nowadays). This book exploits the fact that the Riemannian hypothesis on the metric of the ambient manifold can be sometimes relaxed, thus generalising some well known results such as the existence of the Weierstrass representation for minimal surfaces. From my point of view, this is what singles out this book from the many other works that focus on the Riemannian case.

This book has its origins in two mini-courses given by the author at the Technische Universität of Berlin and at the Federal University of São Carlos. The content has been completed with detailed proofs of the results presented there and many examples. It only assumes from the reader some basic knowledge about differential geometry. Thus the book is rather self-contained, which makes it ideal as a textbook for an advanced graduate course on the subject. The book is well-structured and easy to read. On the other hand, historical references are often overlooked and the results which are either new in this book or in very recent research are not sufficiently highlighted.

The book is composed of six chapters:

- Submanifolds in pseudo-Riemannian geometry.
- Minimal surfaces in pseudo-Euclidean space.
- Equivariant minimal hypersurfaces in space forms.
- Pseudo-Kähler manifolds.
- Complex and Lagrangian submanifolds in pseudo-Kähler manifolds.
- Minimizing properties of minimal submanifolds.

The first chapter contains definitions and basic results on the theory of submanifolds in pseudo-Riemannian manifolds needed in the book, giving either a sketch of their proof or precise references of where they can be found. This will be very helpful to people not familiar with the theory. The special cases of dimension one (i.e. curves)