# Order Out of Chaos

Terry was excited about today's class. In the pre-Calculus class she was teaching, they had begun considering transformations of functions as a way of building larger families of functions. Terry knew that students typically had difficulty visualizing the graphs of functions, but was confident that she could help them learn some basic principles. The day before she had introduced the idea of translations of graphs, using parabolas as examples, and she was looking forward to reinforcing the concepts today. Terry enjoyed teaching this topic because it gave her a chance to show the geometric meaning behind the algebra.

Terry began class by calling for some of the graphing problems assigned the previous day to be put on the board. Natasha and Ralph volunteered to graph $\dfrac{2}{x}$ and $\dfrac{2}{x-3}$ on $[-2,2]$.
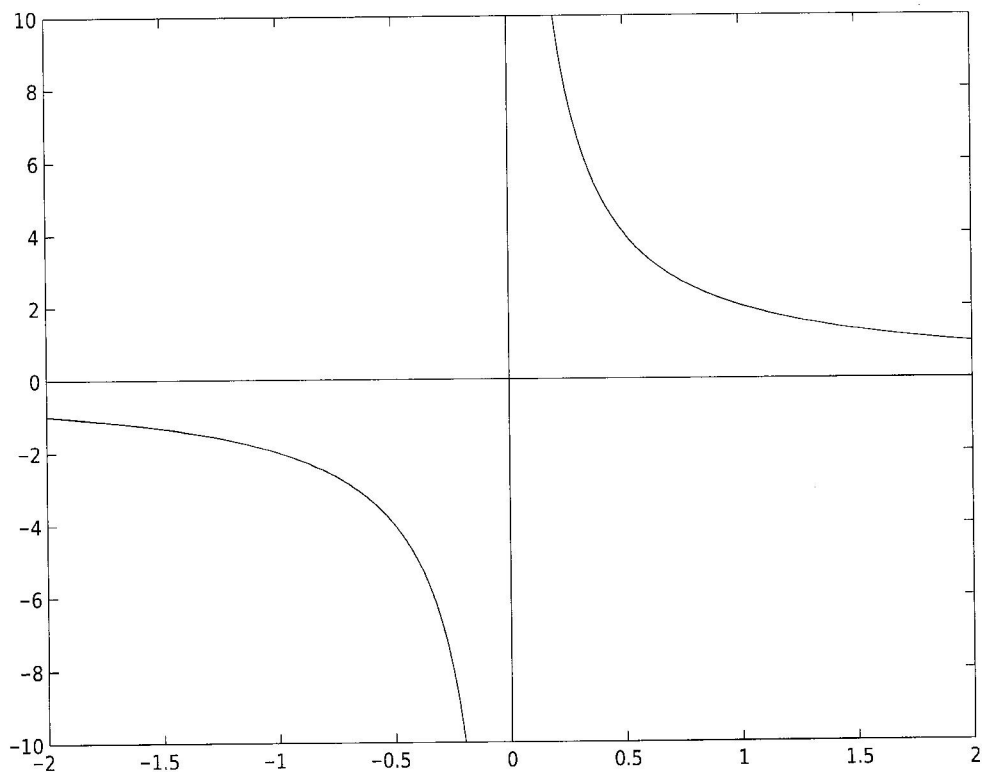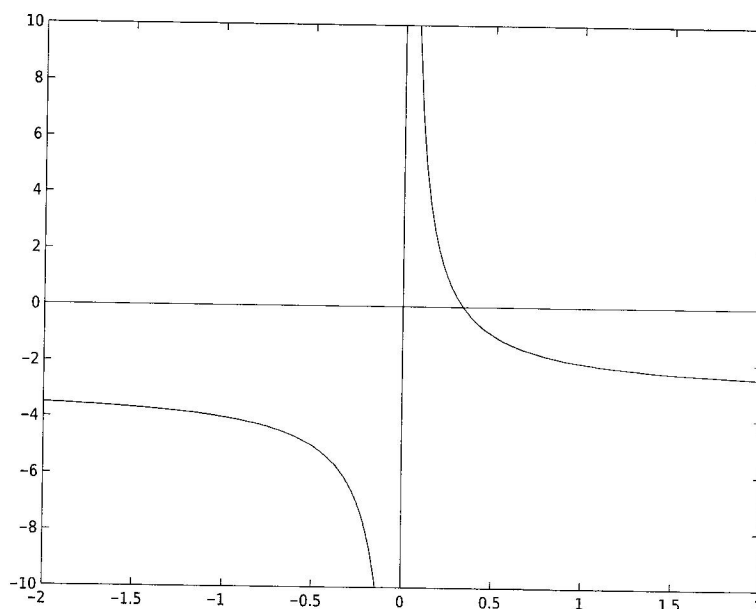

Natasha's graph
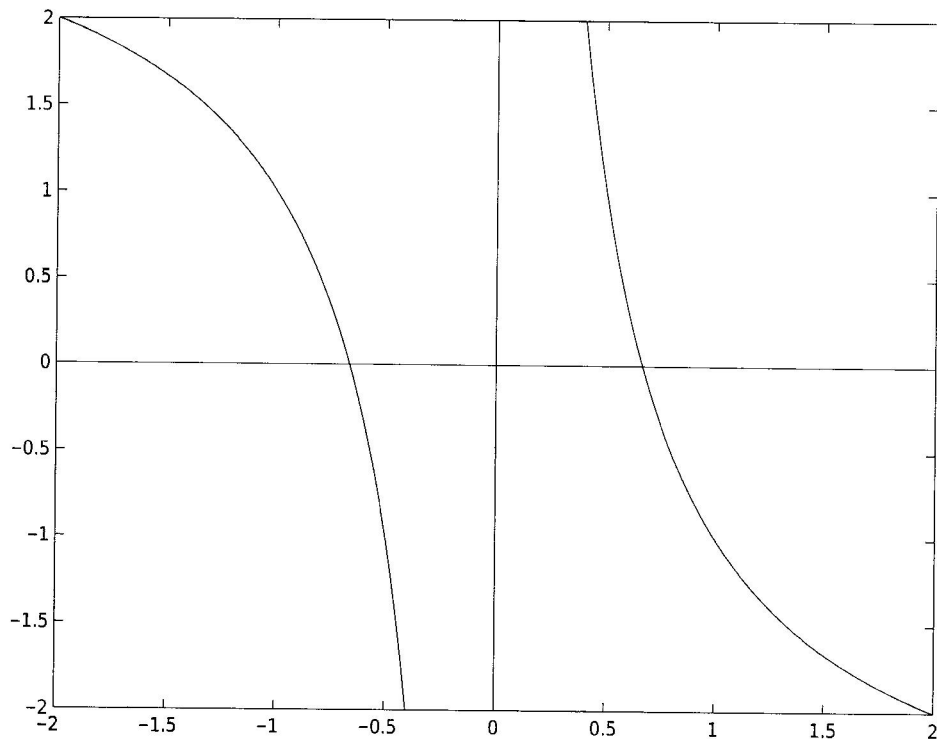
Ralph's graph

Terry looked at the graphs, thought for a few seconds, then turned to the class to ask for comments.

"That's what I got," Bob said. "So did I," added Jane, nodding. Others in the class looked from their papers to the board, saying nothing although some looked puzzled.

Jason announced, "Well I got something different for Ralph's problem!" Terry breathed a silent sigh of relief. "What did you get?" she asked expectantly.

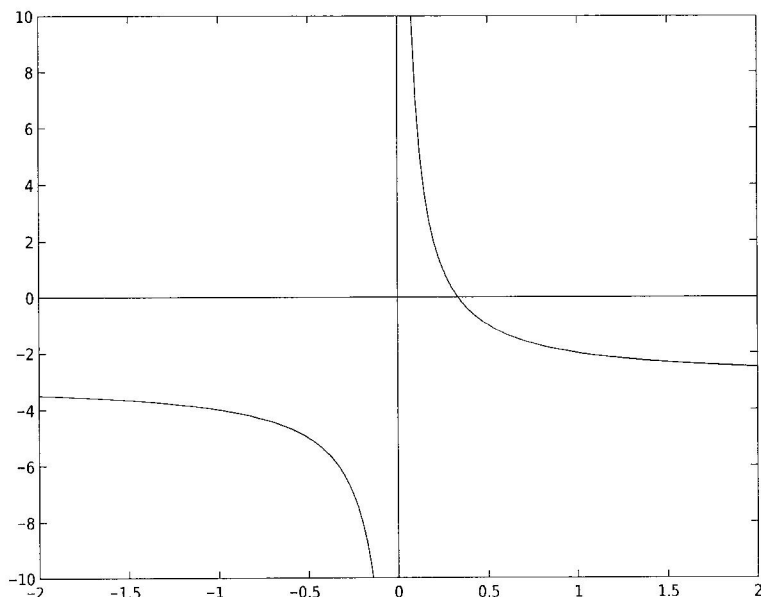Jason went to the board and confidently sketched a quick graph.

Jason's graph

As she watched him working at the board Terry's spirits began to sink again, but she waited until Jason had finished, all the while wondering what to do next. Her original plan to engage the students about the idea of translations seemed to be going awry.

"Ralph, what do you think?" asked Terry. "What did you do to come up with your graph?"

"Well, I first tried out some points on my calculator ..." Ralph looked at his paper, then put up a short table on the board.

1	-1
2	-2
.5	1
.25	5
.1	17

Ralph continued, "And I was going to go on with this, you know plotting points, when I remembered that my roommate had just showed me how to use the graphing feature on my calculator. So I tried it a couple of times, but all I got was the part on the right and nothing was plotted on the left.

Jason's graph

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Ralph continued, "And I was going to go on with this, you know plotting points, when I remembered that my roommate had just showed me how to use the graphing feature on my calculator. So I tried it a couple of times, but all I got was the part on the right and nothing was plotted on the left, so I figured I was just doing something wrong, since there should be some

values for when you plug in negative numbers for x. Anyway, I just sort of 'flipped' the graph over to get my picture, kind of like the $\frac{2}{x}$ picture."

Jane and Bob nodded their heads in agreement, with Jane interjecting that while she had made a table with more points than Ralph, she had also sketched a graph based on her table, and she had done a similar "flipping" procedure. Terry noticed that a few others in the class seemed to be agreeing as well.

Jason, who had been trying to get a word in, finally blurted out, "Yeah, well, I messed around with the graphing feature of my calculator too and figured I needed to stretch out the window in the y direction, since you get big values when x is a little number. I mean look at the last couple of pairs of numbers in your table. Anyway, if you do that - I used $[-10, 10]$ for y - you can see the part of the graph that I got for negative values of x."

Ralph exclaimed, "Oh yeah, I forgot about the window settings. There's a lot to keep track of!"

Returning to the board and picking up the chalk, Jason added, "Besides, if you were going to think about 'flipping' the part of the graph on the right for positive values of x, you'd have to kind of do it around a 'negative 45 degree' line like for $\frac{2}{x}$, except it would have to be shifted down some, sort of like this." Jason drew an additional line on his picture.

Terry glanced at her watch and saw that by now nearly 25 minutes had passed. Somewhat worriedly, she turned to the class and asked for further comments about Ralph's and Jason's work.

After some awkward silence, Marilyn quietly volunteered, "I thought for the second problem that it's like really small, when x is really big, like 300 or something. Natasha's graph is like that, but it doesn't seem to match either Ralph's or Jason's pictures." And from the back of the class Alex asked, "Isn't it just the whole thing shifted to the left..., or is to the right?"

Terry thought to herself, "Well, maybe there is a way to salvage this discussion."