

Fundamental Problems

Keng was the TA for a large calculus class at Western University. Over 80 students attended the professor's lectures 3 times a week, while they had section once a week with Keng in groups of 25-30. Since there was a midterm in two days, Keng had agreed to run a review session for the whole class this evening. He wasn't used to teaching so many students at once, but several students had told him in office hours that they were confused by the professor's lecture on the Fundamental Theorem, and had told him how much they would appreciate a review session before the exam.

Keng began by saying "Okay, this midterm covers Chapter 5, which is about the integral. I think the three most important things we've learned in Chapter 5 are One, what the definite integral is; Two, computing integrals by antiderivatives and the Fundamental Theorem of Calculus; and Three, how to do integrals by substitution. [Keng wrote these three topics on the board.] These are the three topics I suggest you focus on as you review for the midterm. Are there any questions about these topics or problems involving them that you'd like to go over?"

Students began to ask problems. The first was an integral by substitution from the homework. Keng solved it on the board. The second was a standard problem (from the homework and just like one in the text):

A population of bacteria living in a Petri dish has been treated with an experimental drug. If there are 10^6 bacteria initially and the rate of growth at time t hours is observed to be $r(t) = 10^5(1 + t^2)$, find the total number of bacteria in the Petri dish after 3 hours.

As Keng answered this he noticed that the better students were looking bored. As an aside for them, he added, "please notice that we could not solve this problem without the Fundamental Theorem of Calculus. So we see just how important this Theorem is."

Immediately, Lindsey's hand went up. She was a student who was not afraid to speak up, and Keng had noticed that her questions were often useful in figuring out what the class understood and what it did not. Keng called on her.

"I'm really confused about what you just said. In fact, I don't get the whole Fundamental Theorem thing. I thought the integral *was* the antiderivative,

and now we're integrating the growth rate to get the population. I don't see why."

A voice, male, from the back of the room added "Yeah, and what does all this have to do with the derivative of

$$\int_a^x f(t) dt?$$

I don't understand that part at all."

The large room became silent, and Keng wondered what he should say.