Studying the Exam

The following questions have been proposed for a 50-minute midterm. Select the problems that you would use for the exam, giving reasons.

If you feel that there are some important gaps in the problems given, feel free to include problems of your own. If some of the problems need re-wording, feel free to do that also.

In constructing the exam, you should consider the following questions, as well as any others you think important.

- Is the exam the right length?
- Do the problems work out cleanly enough?
- Which questions will be hard to grade? What advice would you give to the graders of each problem?
- Is the exam balanced? In topics? In style of problem?
- Are the instructions to each question clear? Are there any that you think students could misinterpret?
- If the students are using calculators, how will this affect the way in which they approach each problem?
- What order do you think the problems you choose should be in? Do you think it matters?
- How will students do who know only the basics? Will they get some points? Few?
- How will students do who have worked hard and understood most topics, but not all?
- Which problems, if any, rely on students having got the first step right to be able to attempt the main part of the problem? What do you think about such problems?
- Should the students be given a choice of problems on the midterm?

Studying the Exam

Calculus II Questions

The following problems are suggested for a Calculus II exam on techniques of integration and applications of definite integrals to volumes and motion. Calculators with a computer algebra system are not allowed. Your discussion leader will let you know what technology (if any) students will have available.

1. Find
$$\int 2^{\alpha} \cos(3\alpha)\, d\alpha.$$

2. Find
$$\int \theta\, e^{\theta}\, d\theta.$$

3. Find
$$\int 3x^2(1+x^3)^9\, dx.$$

4. Find
$$\int t^3 e^{t^2}\, dt.$$

5. Find
$$\int \sin^3 t \cos t\, dt.$$

6. (a) Calculate $\int_0^2 x^2 - x\, dx$.
(b) Find the area between the curve $y = x^2 - x$, the lines $x = 0$ and $x = 2$, and the x-axis.

7. Find the volume of the region obtained by revolving the region $y = x^2 - x$, with $0 \leq x \leq 2$, around the x-axis. Sketch the region and show your reasoning.

8. Find the volume of the region whose base is the disc $x^2 + y^2 \leq 1$, and whose cross-sections perpendicular to the x-axis are squares.

9. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation

$$\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}} \quad \text{for } t \geq 0$$

with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find

$$\frac{dy}{dt}$$

in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.

(This is a 1998 Advanced Placement® examination problem. Reprinted by permission of the College Entrance Examination Board, the copyright owner.)

10. A particle moves along the curve $y = x^3 - 3x$. The x-coordinate of the particle satisfies the equation

$$\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}} \quad \text{for } t \geq 0$$

and $x = -4$ when $t = 0$. Find the location and speed of the particle when $t = 4$.

11. (a) Show that the total distance traveled by a car moving with velocity, for $t \geq 0$,

$$v(t) = \frac{50}{e^{(\ln 60)t}} \text{ miles per hour}$$

is, in miles,

$$\int_0^\infty \frac{50}{e^{(\ln 60)t}} dt.$$

(b) For $t \geq 0$, does the car travel less than 15 miles in total?