IN MEMORIAM



Remembering Ed Dubinsky and his Visionary Work

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Sadly, Edward (Ed) Dubinsky passed away on May 8, 2022 at the age of 87. He was born in Philadelphia in 1935 to an immigrant family from Kiev, did well in school, and eventually found his way to the University of Michigan. Without a doubt, Ed was one of the founders of the field of research in undergraduate mathematics education (RUME) and one of the most influential RUME researchers to date. Literally, the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME) and the *International Journal for Research in Undergraduate Mathematics Education (IJRUME)* would not exist today had it not been for Ed's preparatory work.

*With input from Maria Trigueros and Kirk Weller.

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Picture of Ed Dubinsky (contributed by J. Dautermann)

Academic Career

In 1962 Ed obtained his PhD in mathematics (differential calculus in Montel spaces) from the University of Michigan. Over the years he was employed by the University of Ghana, Tulane University, the Polish Academy of Sciences, Clarkson University, Purdue University, Georgia State University, and Kent State University, amongst others. From 1963 to 1991, he was a major researcher in functional analysis, publishing 46 papers and supervising seven PhD students¹. From 1983 to 1989, Ed was Director of the Institute for Retraining in Computer Science (IFRICS), which conducted workshops for mathematicians wanting to convert their teaching and research interests to computer science. That was a time when there was a shortage of (and a big demand for) computer science faculty, as there were few computer science PhD programs before 1990.

In approximately 1985, when Ed was a visiting professor at the University of California Berkeley, he began his serious research interest in undergraduate mathematics education. In the subsequent decades, Ed developed APOS (Action-Process-Object-Schema) Theory (see Arnon et al., 2014) and eventually published 74 mathematics education research papers in journals such as *Educational Studies in Mathematics* and *Journal of Mathematical Behavior*. Over this same period, he gave 147 invited lectures and conference presentations, and supervised seven PhD students in mathematics education². Also, for a time beginning in 1985, Ed was active in the Psychology of Mathematics Education's Advanced Mathematical Thinking Working Group, which jointly published a book edited by Tall (1991) in which Ed had a chapter on "Reflective Abstraction in Advanced Mathematical Thinking."

¹Information obtained from the Mathematics Genealogy Project, retrieved June 13, 2022 from https:// www.genealogy.math.ndsu.nodak.edu/id.php?id=5036.

² Ibid.

Ed was influential in the Calculus Reform movement of the 1980s and 90s, writing curricula for calculus (Dubinsky, et al., 1994), abstract algebra (Dubinsky & Leron, 1994), discrete mathematics (Fenton & Dubinsky, 1996) – all were ground-breaking in their use of the programming language ISETL. To support instructors in using the materials, Ed conducted several National Science Foundation (NSF) funded workshops for college faculty and contributed to a guide on using cooperative group learning (Hagelgans et al., 1995). Throughout its run, from 1989 to 1996, Ed edited the newsletter *UME Trends: News and Reports on Undergraduate Mathematics Education*, an NSF-sponsored joint venture of the American Mathematical Society (AMS), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM), which was considered part of the Calculus Reform movement in the United States during the late 1980s and early '90s (Dubinsky, April 1996).

As Annie Selden later recalled,

When John and I interviewed him, at the time, on how he prepared for his college faculty workshops on teaching with his calculus and abstract algebra curriculum materials, Ed told us that he went over every detail "like a movie" in his head in preparation for these workshops. Ed was not a man to leave important matters to chance.

The NSF-funded curriculum projects resulted in textbooks, publications, and presentations, paving the way for the development of RUME as a legitimate research field. As Ed himself noted, this took a lot of persistent unsupported preparatory work,

for some years there were rules in NSF against *any* support for post-secondary mathematics education. Many of us began working in this area in spite of the lack of any *possibility* of support. When this changed and NSF support for calculus reform began and research in teaching and learning was opened up to mathematical topics beyond high school, it was not a horde of newcomers that received the first grants, but mainly people who had been working for several years, with no support at all (Dubinsky, February 1996).

APOS Theory and RUMEC

Perhaps most importantly, Ed is well known for his development of APOS theory, an extension of Piaget's theory of reflective abstraction to the learning of undergraduate mathematics. Now widely used, the related ACE (Activities, Class discussion, Exercises) teaching cycle is an instructional approach that supports development of the mental constructions called for by APOS theory.

APOS Theory focuses on models of what might be going on in the mind of an individual when he or she is trying to learn a mathematical concept and uses



Fig. 1 Diagram representing the connections in mathematical mental activity – from considering actions on objects, interiorizing those actions into coordinated processes and using encapsulated processes as new objects on which to consider actions and, conversely, de-encapsulating objects into component processes for further refinement and subsequent, purposeful, (re)encapsulation

these models to design instructional materials and/or evaluate student successes and failures in dealing with mathematical situations (Arnon et al., 2014, p. 1).

When asked about his theory, Ed would often show the diagram in Fig. 1 (taken from his home page at Kent State University at http://www.math.kent.edu/%7Eedd/).

In 1995, Ed founded the Research in Undergraduate Mathematics Education Community (RUMEC). The focus for this group of college teaching faculty was to conduct research studies using APOS theory (Dubinsky, 1997). This was elaborated on by Draga Vidakovic, Ed's former PhD student, collaborator, and one of the founding members of the RUMEC group:

The initial focus of RUMEC was to learn the basics of APOS and use it in analyzing a large amount of data about students learning calculus, abstract algebra, and discrete mathematics collected during several NSF-funded projects. Later, the RUMEC group worked on tuning and expanding the APOS Theory. The work of RUMEC was funded by The Exxon Educational Foundation from its inception till the founding of ARUME, at which point the funding was transferred to this group. The funding of RUMEC was used for two or three group meetings per year (D. Vidakovic, personal communication, June 13, 2022).

Further elaborating on RUMEC, Maria Trigueros, also a member of the group, noted that.

Ed's work with the community of researchers that was called RUMEC was fundamental in creating a rich environment where collaborative work and rich discussions were central. Those who had the opportunity to be part of the group formed a very friendly collectivity where research studies were seriously discussed and where the group of authors had total freedom to continue working on the study. This methodology resulted in the publication of many APOS theorybased research papers and helped the RUMEC community grow as researchers. The same can be said of Ed's role during the work on the APOS theory book (Arnon et al., 2014). He contributed as an equal with the other authors, promoting critical but respectful discussions. The creation of a productive and friendly environment made the publication of the book possible in a short period of time. (M. Trigueros, personal communication, June 16, 2022).

ED DUBINSKY

You have been a man of ACTION in RUME and encouraged us to interiorize your actions into a PROCESS for doing RUME. This PROCESS was encapsulated into an OBJECT called RUMEC. But even that was not enough as you coordinated the development of a schema for RUME called ARUME that has now been thematized into an OBJECT on which we will continue to perform ACTIONS. With appreciation from your RUMEC colleagues February 26, 2000

Ed's birthday plaque - gift from RUMEC



RUMEC members celebrating Ed's 80th birthday, February 2015^{3***}

The Story Behind the SIGMAA on RUME and IJRUME

An organization such as the SIGMAA on RUME, and a journal such as *IJRUME*, do not appear full-blown overnight—there needs to be a lot of preparatory work (see Appendix for a timeline of events). Many of the following details first appeared in a report on the prehistory of the SIGMAA on RUME (Selden, 2012). According to Ed's recollection, in the early 1990s not many people with strong mathematics backgrounds were interested in RUME. Mathematics education research journals of the day were not anxious to publish papers in RUME. Also, no mathematics departments were interested in having faculty members doing such research. Ed set out to change that.

Three major developments followed. The first major development was that Ed organized, along with Jim Kaput and Alan Schoenfeld, a Special Session on Research in Undergraduate Mathematics Education at the 1991 Joint Mathematics Meetings (JMM), inviting mathematics education researchers from around the world to speak. And, though the Mathematical Association of America (MAA) was not interested in forming a special interest group in 1992, it was amenable to forming an AMS/MAA Joint Committee on Research in Undergraduate Mathematics Education (CRUME), with five members each from the American Mathematical Society (AMS) and MAA, and one member each from the National Council of Teachers of Mathematics (NCTM) and the American Mathematical Association of Two-Year Colleges (AMA-TYC), with Ed as its first chair.

³*** Front row: Ed Dubinsky, Ilana Arnon.Back row, from left to right: Bernie Baker, Maria Trigueros, Kirk Weller, Draga Vidakovic.

The second major development, one of the first projects of CRUME, was the establishment, under the Conference Board of the Mathematical Sciences of a book series on RUME, called *Research in Collegiate Mathematics Education (RCME)*. Ed Dubinsky, Alan Schoenfeld, and Jim Kaput were the founding editors. Over the years, seven volumes were produced, with other editors beginning with *RCME V*. From the beginning, the idea was to demonstrate that there was enough high-quality research in undergraduate mathematics education to warrant a journal. Work to establish a venue for publication of RUME was continued by others and finally came to fruition with the first issue of *IJRUME* in April 2015 under founding editors Karen Marrongelle, Chris Rasmussen, and Michael O. J. Thomas. The full story of the many attempts and failures to get a journal for RUME deserves an article of its own. However, it should be noted that it took 23 years for Ed's (and many others') hope, work, and intervening disappointments before the journal *IJRUME* appeared.

The third major development was the work towards a special interest group for RUME within the MAA. Initially the MAA was not receptive to the idea of special interest groups for fear it would splinter the organization. However, eventually in January 1999, ARUME (the Association for Research in Undergraduate Mathematics Education) was formed at the Joint Mathematics Meetings with a business meeting followed by a reception, funded in part by the Exxon/Mobil Foundation. The following year, in 2000, this organization became the first special interest group of the MAA. Many more MAA Special Interest groups (SIGMAAs) followed over the years (as of this writing, there are 17 of them, MAA, 2022).

Several RUME community-building and dissemination efforts paralleled these major developments. In the early 1990s, before the formation of ARUME, came the publication of two MAA Notes Volumes on research in undergraduate mathematics education. The first was *The Concept of Function: Aspects of Epistemology and Pedagogy*, edited by Ed Dubinsky and Guershon Harel (1992), which is still quoted today. The other was a subsequent volume, *Research Issues in Undergraduate Mathematics Education: Preliminary Analyses and Results*, edited by Jim Kaput and Ed Dubinsky (1994).

Another important activity of CRUME to gain acceptance of RUME by mathematicians was the establishment of a *Mathematical Reviews* subject classification, 97 Mathematics Education, and subclassifications. This subject classification enabled mathematicians and mathematics education specialists applying for academic and other jobs to list 97 on their AMS Cover Sheets. Unfortunately, at the time, no money to implement actual reviews was appropriated. However, to understand the importance of just the establishment of this subject classification, here is how David Tall recalls the situation:

... a major problem arose because mathematics education research was not accepted as an academic track in university mathematics departments. The London Mathematics Society refused to allow me to state my area of interest as "mathematical thinking" because it did not appear in the American Mathematics Association Subject Classification. I corresponded with Ed and he took up the question with the AMS and actively pursued it until, after a long and wind-

ing road, "pedagogy of mathematics teaching" became subject 97 in the AMS classification (D. Tall, personal communication, July 15, 2022).

In addition, before conferences became an activity of the SIGMAA on RUME, Ed and the RUMEC group organized three conferences on research in undergraduate mathematics education: the first two were held in September, 1996 and 1997, in Mt. Pleasant, Michigan. The third was held in September, 1998 in South Bend, Indiana. These three early, well-received conferences were an attempt to convince the MAA leadership of the support for, and interest in, research in undergraduate mathematics education as the basis for a professional network. After the formation of SIGMAA on RUME, Ed advised the RUMEC group to "pull back from these roles ... He strongly believed that delegating the roles to other RUME members was the best way to grow and strengthen SIGMAA on RUME" (D. Vidakovic, personal communication, June 13, 2022).

Ed was very persistent over many years in his efforts to put research in undergraduate mathematics education on the map. He battled to get mathematicians to see the value of such research, to get it published, and to provide a professional home for it (see timeline in Appendix). As Draga Vidakovic recently wrote,

Ed strongly believed that high quality work, and high-quality publications were the ways to convince mathematicians in the value of research in collegiate mathematics education. He emphasized this in all our work within RUMEC. Motivated by this, the RUMEC group established an activity that was called "internal review", a designated period of time at the RUMEC group meetings during which the entire group discussed/reviewed a paper written by a subgroup of RUMEC that was ready to be submitted for publication. The paper was distributed and read by the members prior to the meeting. We believed that this particular activity had two main accomplishments, one was a production of much improved research report, and the second was researchers' growth in this kind of activities.

Needless to say, Ed was very open and critical during the internal review. We all had to learn not to take his criticism personally but rather as his desire to help us improve as researchers and subsequently produce high quality research reports. On the other hand, very often, Ed was much further in his thinking than the rest of us, especially at the beginning of RUMEC work. In such situations, he was patient to discuss the issues till we understood his thinking and reasoning (D. Vidakovic, personal communication, June 13, 2022).

Ed once spoke to Annie and John Selden of his approach to curricula and its teaching as essentially providing a "holistic spray" of activities, concepts, exercises, and theory. If one looks back on Ed's persistent efforts over the years, it seems that he was also using the technique of holistic spray on behalf of RUME.

Personal Reflections of Ed

The following is a collection of personal remembrances of Ed from a variety of colleagues who knew and worked with Ed over the years, including Uri Leron with whom he wrote an ISETL-based abstract algebra textbook (Dubinsky & Leron, 1994); Rina Zazkis, with whom he coauthored a *JRME* article (Zazkis et al., 1996); and David Tall, with whom he had differences of opinion on how to incorporate technology into mathematics education.

Ed was not a man to suffer fools gladly and was always well aware of his rights. He told Annie and John Selden that he once went to an AMS Committee meeting, was asked to leave, and told the assembled members of that AMS Committee that such meetings were open to all AMS members and refused to leave—a fact of which he seemed justly proud.

Ed was passionate about social justice and certain related causes. Early on when Ed was a researcher in functional analysis, at the annual Joint Mathematics Meetings, he lobbied and petitioned for the release of imprisoned Soviet mathematicians. When he was a faculty member at Tulane, he was involved in anti-Vietnam War demonstrations, which led to his dismissal from that university and the resulting time from 1970 to 1972 that he spent at the Polish Academy of Sciences. Alan Schoenfeld recalled.

In the 1960s, Ed's vehement and vocal protests against the Viet Nam war and against civil rights injustices in the South made him a political hot potato – he said that his politics at that time made him close to unemployable. Through the latter 20th century, he was involved in charity donations of math textbooks to Cuba. Much later, his admiration for civil rights icon Bob Moses, who died in July 2021, led Ed to join forces with Bob and work on curriculum development for the Algebra Project (A. Schoenfeld, personal communication, July 16, 2022).

In this regard, Jennie Dautermann, a retired English professor and Ed's life partner, recalled.

At Tulane, Ed learned about "real jazz" and spent his weekends in nearby Laurel, Mississippi organizing voter registration projects, and sponsoring several student groups protesting the Viet Nam war back at Tulane during the week in addition to his teaching duties. These groups frequently heckled the student soldiers as they practiced their army drills. After some pretty graphic news coverage, and objections to his other anti-war protests on campus, Ed was brought up on charges and fired. Unable to find a similar job anywhere in the US, he found work in Europe where ideas about teaching and learning as well as approaches to Mathematics challenged him to think in new and fresh ways (from Ed's obituary sent by J. Dautermann, June 12, 2022).

Collaborations with Colleagues

Uri Leron, Professor Emeritus, Department of Education in Science and Technology, Israel Institute of Technology, recalled his first meeting with Ed:

First, sometime in the 80s (or was it the 90s?) Ed arrived in Israel for the annual PME conference. After the conference, he came to visit me at the Technion to try to recruit me into the ISETL community. I had been then very much into working with Logo, and couldn't imagine that the esoteric language ISETL, which was completely unknown at the time, could have anything similar to offer. Ed tried to convince me that ISETL was particularly suitable for learning college-level math, and eventually I said to him: Ok, show me how you define a group in ISETL. ("define a group" in computational terms meant, write a procedure that inputs a set and a binary operation and returns true or false according to whether the input parameters form a group or not.) Ed walked over to the board and scribbled:

```
is_group := func(G,o);
return.
is_closed(G,o) and
is_associative(G,o) and
has_identity(G,o) and
has_inverses(G,o);
end;
where, for example:
is_closed := func(G,o);
return.
forall a,b in G | a .o b in G;
end;
```

I was greatly impressed by the similarity to the mathematical definition (which ensures only a small programming overhead when teaching the algebra course; try in comparison to express the group concept in Logo or even LISP), and our collaboration started there and then. (U. Leron, personal communication, June 20, 2022).

Of their time together writing the abstract algebra textbook (Dubinsky & Leron, 1994), Uri Leron recalled that they were

taking turns writing the first draft of each chapter, then the other one would read and comment and, finally, we would meet and work on the final version, arguing away the differences of opinion. What happened there was a small miracle, considering the fact that both of us were highly experienced abstract algebra instructors with strongly opinionated personality. Instead of hotly arguing our individual (and often conflicting) opinions, we would each state our position, and then add *a quantitative estimate of how much we cared*. For example, I could say, I think differently but I only care 40%, and if Ed weighed his position at 60%, we would adopt it. The small miracle was that his choices and mine came out reasonably balanced in "winning out," so we were both satisfied with the process (U. Leron, personal communication, June 20, 2022).

Amongst Ed's many contributions, mentioned by colleagues, there is one not often highlighted: that of mathematician and teacher of mathematics. Rina Zazkis Professor, Faculty of Education, Simon Fraser University, contributed this remembrance,

Many years ago, working with dihedral groups,

I experienced a mathematical disequilibration – something did not fit with my (and conventional) understanding of functions and transformations.

In considering composition of isometries of a square.

(f * g) (x).

things actually "worked" when applying f before g,

rather than the conventional interpretation as.

"f applied on the result of g(x)"

f[g(x)]

By "worked" I mean consistency with carrying out the transformation on the physical square.

I approached several colleagues, who totally dismissed my problem, either implying that I miscalculated (I did not!) or saying that it is "OK" in this case to compose function by ignoring the conventional order.

It was ED AND ONLY ED, who acknowledged the discrepancy, sought a solution, and found a reference, in an old algebra book, where the "problem" with dihedral groups that I experienced was actually acknowledged and explained. The "problem" is likely familiar for those researching Abstract Algebra. For others.

I invite you to consider rotation of a square 90 degrees clockwise, where the vertices and their positions are labeled 1-2-3-4 clockwise.

What should it be?

 $(1234) \Longrightarrow (2341).$



A solution and a discussion can be found in Zazkis & Dubinsky (1996). In addition to all the other great things that Ed has done, he enlightened my mathematics. (R. Zazkis, personal communication, July 15, 2022).

Further Reflections

David Tall, Professor Emeritus, The University of Warwick, who over the years had a sometimes contentious relationship with Ed, recalled his initial meeting with Ed was about using technology in mathematics teaching, a common interest, which was expressed in very different ways.

Ed Dubinsky was undoubtedly a leading figure in the introduction of technology in mathematics teaching in the latter half of the twentieth century. He and I first met in Strasbourg in 1985 at the ICMI conference on computers and informatics. At the time he was using the text-based programming language ISETL which he had designed to write quantified mathematical statements to seek solutions. I had published my own graphical programs to draw dynamic pictures of the processes of differentiation, integration and solving differential equations.



Ed Dubinsky and David Tall at PME 12 in Hungary, 1988. (photo contributed by David Tall)

Although both David Tall and Ed were influential in introducing technology into mathematics teaching, their approaches diverged. As David Tall recalled,

We had very different perspectives. ... Ed sought to set the agenda for future development of undergraduate mathematical thinking in which his views of reflective abstraction, cooperative learning and ISETL played a leading part. At a conference that he invited me to attend, he spoke about the use of ISETL to encourage mathematical thinking. I commented that each computer language had its own structure and could not incorporate all possible ways of thinking mathematically. As an example, I suggested that I could think of a set S that contained a variable quantity x but, if I defined a variable x to belong to S, then in ISETL the set S would only contain the *numerical value* of x at the time of input.

For certain, my personal growth was greatly enhanced by my interactions with a driven, sometimes infuriating, always dedicated, Ed Dubinsky (D. Tall, personal communication, July 15, 2022).

Often Ed did not want to take major credit for the mathematics education research papers written jointly by members of the RUMEC group. He told Annie and John Selden that he thought it was fair and appropriate to list the authors of such joint papers alphabetically (e.g., Asiala et al., 1996). Indeed, this was true for most RUMEC papers regardless of whether Ed was an author or not. At least, this was true for the first few years of RUMEC existence.

Anne Brown, Professor of Mathematics Emeritus, Indiana University South Bend, one of the founding members of the RUMEC group and a co-author of several papers with Ed, had the following reflections:

I met Ed in 1995 when I participated in his summer workshop on cooperative learning at Purdue University. As a side project at this workshop, he offered mathematicians an opportunity to learn to conduct research in collegiate mathematics education.

Ed needed help analyzing a large set of student data from ongoing research projects on the learning of advanced mathematics. A group of us, under Ed's leadership, formed the collaborative research group RUMEC. Several subgroups of RUMEC members worked to learn the APOS framework, analyzed the data from the projects, and wrote reports for publication. Working collaboratively on research was a new and welcome experience for me and I felt I had finally found a scholarly home.

As a mentor, collaborator and friend, Ed was unfailingly generous with his time and knowledge. His leadership, through RUMEC, helped me shape the remainder of my career. I appreciated the discipline of adhering to a single theoretical framework (APOS) that resonated strongly with my experience as a mathematician. The internal reviews we did of our fellow members' research taught us how to give and receive constructive criticism, and helped us grow as researchers and writers. Serving as a leader in RUMEC myself helped me become a more effective faculty member and organizer, which enhanced my service to my university. And most significantly, I became a more reflective, deliberate and creative teacher in the classroom.

For these reasons, I always think of my time working with Ed with profound gratitude, and I am sure that many other RUMEC members feel the same way.

Ed remained true to APOS Theory. He did not look kindly on the theoretical and methodological pluralism favored by today's researchers. Once, at an early RUME Conference, Ed asked Shandy Hauk, now a Professor of Mathematics at San Francisco State University, whether she had students presenting papers at the conference. When she replied three students, one using APOS Theory, one using Pirie and Kieren's mathematical thinking framework, and one using Bandura's theory of self-efficacy, Ed waved her away saying she was a "theoretical butterfly"—a characterization she has treasured ever since (S. Hauk, personal communication, June 24, 2022).

Appendix

Timeline of Ed's involvement in preparatory events leading to the SIGMAA on RUME and *IJRUME*.

1985-86 Ed spends time at U.C. Berkeley developing APOS theory.

1989 Establishment of the NSF-funded newsletter, *UME Trends: News and Reports on Undergraduate Mathematics Education*, an NSF-funded joint venture of AMS, MAA, and SIAM, with Ed serving as Editor. This publication was discontinued in 1996.⁴ In recognition of his contributions, Ed received a Certificate of Recognition from the MAA at its annual meeting in Orlando in 1996.

1990 Purdue Conference on Learning Functions, resulting in the MAA Notes volume, *The concept of function: Aspects of epistemology and pedagogy* (Dubinsky & Harel, 1992).

1991 Special Session on Research in Undergraduate Mathematics Education at the JMM organized by Ed Dubinsky, Jim Kaput, and Alan Schoenfeld.

1992 Formation of the AMS/MAA Joint Committee on Research in Undergraduate Mathematics Education (CRUME).

1994 Publication of MAA Notes volume, *Research issues in undergraduate mathematics education: Preliminary analyses and results*, (Kaput & Dubinsky, 1994)

1994 Publication of *Research in Collegiate Mathematics Education. I*, by CBMS, a project of CRUME, the first of seven such volumes, with the final one appearing in 2010.

1995 Formation of the Research in Undergraduate Education Community (RUMEC) for mathematicians interested in doing research using APOS Theory.

1996, 97, 98 First three annual *Conferences on Research in Undergraduate Mathematics Education*, organized by Ed and RUMEC in Mt. Pleasant, Michigan and South Bend, Indiana in September of each year.

⁴ This was said to be due to the expansion of mathematics education articles in MAA periodicals. (Daniels & Tucker, 1996).

1999 Initial meeting at JMM, under the auspices of CRUME, to form the Association for Research in Undergraduate Mathematics Education (ARUME).

1999 Fourth annual *Conference on Research in Undergraduate Mathematics Education*, organized by ARUME and held in Rosemont, Illinois.

2000 Fifth annual Conference on Research in Mathematics Undergraduate Education, organized by ARUME and held in Chicago, Illinois.

2001 The Special Interest Group of the Mathematical Association of America on Research in Undergraduate Education (SIGMAA on RUME) was formed at the JMM as its first special interest group.

2001 The sixth annual *Conference on Research in Undergraduate Mathematics Education*, the first conference organized by the SIGMAA on RUME was scheduled to be held in Chicago, Illinois in September, but was postponed due to the 9/11/2001 terrorist attacks.

2002 Finally, the sixth annual *Conference on Research in Undergraduate Mathematics Education*, organized by the SIGMAA on RUME, was held in Burlington, Vermont in Summer 2002. Over the years, Ed became less involved in the SIGMAA on RUME and its annual conferences, but continued his research full steam with RUMEC group members, using APOS Theory.

2005 CRUME disbanded as its work was done, a fact of which Ed was proud. Also, in that year, the RUMEC group was disbanded as, by that time, many of the RUMEC members were experienced researchers and participated fully in the SIG-MAA on RUME. However, subgroups of people from RUMEC continued to collaborate on various project till the present time. Ed participated in many of those projects.

2015 First issue of *IJRUME* appears with initial editors Karen Marrongelle, Chris Rasmussen, and Michael O. J. Thomas⁵.

2018 Ed was honored at a special session the first day of the twenty-first annual *Conference on Research in Undergraduate Mathematics Education*, held in San Diego California.

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⁵ The full story of the many attempts and failures to get a journal for RUME (after the publication of the seven *RCME* volumes) involved many others, including various subsequent officers and members of the SIGMAA on RUME, and deserves an article of its own.

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