Making the Grade (Calculus I Version)

You, as the teaching assistant, are working with a class of students taking Calculus I. Grade the sample student work below first on a 10-point scale, as if these were problems on a quiz or an exam, and then on a 3-point scale, as if these were problems on the homework. Write your scores next to the student work in the appropriate column.

1. Find the derivative of $y = \sec^2(1 + 3x)$.

10-pt scale 3-pt scale

Student A:

$y = (\sec u)^2 \quad u = 1 + 3x$

$y' = 2(\sec(1+3x))(\sec x \tan x) \cdot 3$

$= 6 \sec x \tan x \sec(1+3x)$

Student B:

$y' = 2 \cdot 3 \sec(1+3x) \tan(1+3x)$

$= 6 \sec(1+3x) \tan(1+3x)$

Student C:

$y = \sec(1+3x) \cdot \sec(1+3x)$

$\sec \tan(1+3x) \cdot 3 \cdot \sec \tan(1+3x) \cdot 3$

$\boxed{18 \sec \tan(1+3x)}$

Student D:

$y' = \tan(1+3x) \cdot 3$

2. Let $f(x) = \sqrt{2x^2 - 4}$. Find $\lim_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

10-pt scale 3-pt scale

Student A:

$$\lim_{x \to 2} \frac{\sqrt{2x^2-4} - \sqrt{2(2)^2-4}}{x-2}$$

$$\lim_{x \to 2} \frac{\sqrt{2x^2-4} - 2}{x-2} = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$$

Student B:

$$f(x) = \sqrt{2x^2-4}$$

$$f(2) = \sqrt{2(2)^2-4} = \sqrt{8-4} = 2$$

$$\lim_{x \to 2} \frac{\sqrt{2x^2-4} - 2}{x-2} \cdot \frac{\sqrt{2x^2-4} + 2}{\sqrt{2x^2-4} + 2}$$

$$= \lim_{x \to 2} \frac{2x^2 - 4 - 4}{(x-2)(\sqrt{2x^2-4}) + 2(x-2)}$$

$$= \lim_{x \to 2} \frac{2x^2}{(x-2)\sqrt{2x^2-4} + 2x^2 - 4}$$

$$= \frac{2(2)^2}{0 + 8 - 4} = \frac{8}{4} = 2$$

Student C:

$$\lim_{x \to 2} \frac{\sqrt{2x^2-4} - 2}{x-2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 2} \frac{(2x^2-4)^2 - 2}{x-2}$$

$$\overset{L'H}{=} \lim_{x \to 2} \frac{2(2x^2-4) \cdot 4x}{1} = 2(8-4)\,8$$

$$= \boxed{64}$$