(1) Construct a closed, uncountable subset of $[0,1]$ that contains no rational numbers.

(2) If $E \subset \mathbb{R}$ is closed, let $E' \subset E$ be the set of its limit points (non-isolated points). Is there a set E so that $E, E', E'', \dots$ are all distinct and non-empty?

(3) Construct a continuous function $f : [0,1] \to [0,1]$ that takes every value uncountable often.

(4) Does the set of continuous functions $f : [0,1] \to [0,1]$ have the same, smaller or greater cardinality than $[0,1]$?

(5) Give an example of a subset $E \subset [0,1]$ that is a countable union of closed sets, but is not a countable intersection of open sets.