as a parametric solution of (10). Hence from (9), taking the plus sign before α,

$$a_1 = 7m^2 + 13mn - 30n^2.$$

Then from (8), $a_2 = 13m^2 - 22mn - 26n^2$. Finally from (5),

$$a_3 = -8m^2 + 39mn - 16n^2, \qquad b_3 = -13m^2 + 24mn - 26n^2.$$

The negative sign before α only interchanges a_1 and a_3 with sign changed. If we denote the quadratic form $am^2 + bmn + cn^2$ by the notation $[a, b, c]$, we write the solution of the system (3) as

$$a_1 = [7, 13, -30], \qquad a_2 = [13, -22, -26], \qquad a_3 = [-8, 39, -16]$$
$$b_1 = [-7, 13, -16], \qquad b_2 = [8, -13, -30], \qquad b_3 = [-13, 24, -26].$$

By Theorem 3, the system (2) has then the following parametric solution:

$$A_1 = [-7, 62, -30], \qquad A_2 = [7, 38, -50], \qquad A_3 = [5, -8, -22],$$
$$A_4 = [19, -32, -42], \qquad A_5 = [-19, 36, -62], \qquad B_1 = [-9, 66, -42],$$
$$B_2 = [5, 42, -62], \qquad B_3 = [-21, 38, -22], \qquad B_4 = [9, -14, -50],$$
$$B_5 = [21, -36, -30].$$

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PROJECTING m ONTO c_0

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It is a well-known result, due to Phillips, that the Banach space m, of bounded sequences with the sup norm, cannot be projected continuously onto the subspace c_0 of sequences converging to zero [1, page 33, Corollary 4]. A typical use of this fact is found in [2]. We give a simple proof using an idea inherent in [4] and, as was pointed out by the referee, in [3]. Our method may also be used to simplify the proof of the result in [4].

LEMMA [5, page 77]. *Let I be a countable set. Then there is a family $\{U_a: a \text{ in } A\}$ of subsets of I such that* (1) *U_a is infinite,* (2) *$U_a \cap U_b$ is finite for $a \neq b$ and* (3) *the index set A is uncountable.*

Proof. Arthur Kruse has given the following elegant proof: Take I to be the rationals in (0, 1), A the irrationals in (0, 1) and, for a in A, let U_a be a sequence of rationals in (0, 1) converging to a.

Recall that a subset of the conjugate space X^* of a Banach space X is total if the only vector annihilated by all members of the subset is the zero vector.

For brevity we say that a Banach space X has (property) B if X^* contains a countable total subset. It is easy to see that B is preserved under isomorphism, that a subspace of a space with B has B and that the space m has B.

THEOREM. *There is no continuous projection of m onto c_0.*

Proof. Suppose that there is a continuous projection of m onto c_0. Then $m = c_0 \oplus R$, where R is a closed subspace of m. Since m/c_0 is isomorphic to R we see that m/c_0 has B. The proof consists of showing that m/c_0 does not have B.

We think of m as $B(I)$, the bounded functions on a countable set I. Let $\{U_a: a \text{ in } A\}$ be a family of subsets of I as in the lemma and let f_a be the coset in m/c_0 which contains the characteristic function of the set U_a.

Let g be in $(m/c_0)^*$. We will show that the set $\{f_a: g(f_a) \neq 0\}$ is countable; it suffices to show that the set $C(n) = \{f_a: |g(f_a)| \geqq 1/n\}$ is countable for each natural number n. Choose $f_1, \cdots, f_m$ in $C(n)$ and let $b_i = \text{sgn}(g(f_i)) = \overline{g(f_i)}/|g(f_i)|$. The vector $x = \sum b_i f_i$ is of norm one (note that as a coset x contains vectors whose norm may be greater than one), and so $\|g\| \geqq |g(x)| \geqq m/n$; thus $C(n)$ is finite for each n.

We conclude by noting that if $\{h_i\}$ is a countable subset of $(m/c_0)^*$ then our argument shows that there are only countably many f_a with $h_i(f_a)$ nonzero for some i. Hence we can find a vector f_a which is mapped into zero by all the h_i, and so the set $\{h_i\}$ is not total.

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INTERIORITY AND THE TONELLI CONDITIONS

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In 1937, S. Stoïlow proved that if f is a complex-valued function of a complex variable which has the properties: (i) point inverses are totally disconnected, and (ii) f maps interior points of its domain of definition into interior points of the image, then f is topologically equivalent to an analytic function. This result stimulated interest in light interior functions (i.e. functions satisfying (i) and (ii)) and in establishing conditions which insure that a function satisfying these conditions will be light and interior. Titus and Young proved that if $f \in C'$ and