MAT 342 Fall 2016, SAMPLE MIDTERM 1,
Actual midterm is 10:00-10:53am, Friday, October 14, 2016

Name	ID	

THIS EXAM IS WORTH 50 POINTS. EACH QUESTION I SWORTH ONE POINT. NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED.

1-10 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(1) T $(2-3i)-(4+2i)=-2-5i$

(2) T $(2+i)(3+i)=5+5i$

(3) F $(1-i)^3=-1-i.$

(4) F $1/i=i$

(5) F $e^{\pi i/4}=\sqrt{2}(1+i)$

(6) T $e^{100\pi i}=1$

(7) F $\frac{i}{2-i}=\frac{1+2i}{3}$

(8) F $\mathrm{Log}(-1)=\pi$

(9) T $\arg(1+i)=\{\frac{\pi}{4}+2\pi n : n\in\mathbb{Z}\}$

(10) T $e^{i}=\cos(1)+i\sin(1).$

11-15 Place the letter of the corresponding point in the box. The same letter might be used more than once.

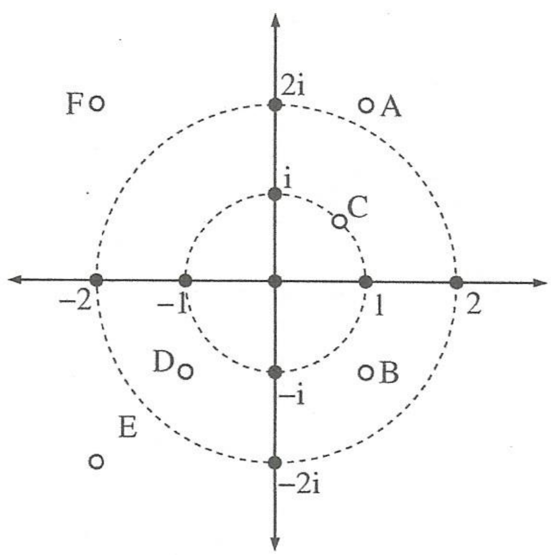
(11) A $|z|=\sqrt{5}$

(12) D $\mathrm{Re}(z)=-1.$

(13) B $z^2=-2i$

(14) E $z=\overline{F}$

(15) B $\mathrm{Arg}(z)=-\pi/4.$



16-20 Match each function with its definition. Assume $z = x + iy$.

(16) $\boxed{K}$ $\sinh(z)$

(17) $\boxed{I}$ e^z

(18) $\boxed{A}$ $\sin(z)$

(19) $\boxed{C}$ $\tan(z)$

(20) $\boxed{J}$ i^z

A. $\frac{1}{2i}(e^{iz} - e^{-iz})$

B. $\frac{1}{2}(e^{iz} + e^{-iz})$

C. $(-i)\frac{e^{iz}-e^{-iz}}{e^{iz}+e^{-iz}}$

D. $\frac{e^{iz}+e^{-iz}}{e^{iz}-e^{-iz}}$

E. $\frac{1}{2}(e^z + e^{-z})$

F. $e^y(\cos x + i \sin x)$

G. $e^x(\cos x - i \sin x)$

H. $e^x \cos(y)$

I. $e^x \cos(y) + ie^x \sin(y)$

J. $e^{z \log i}$

K. $\frac{1}{2}(e^z - e^{-z})$

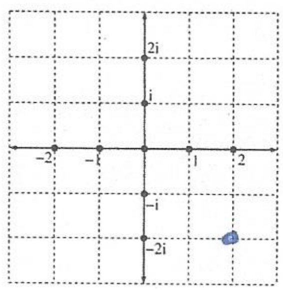
L. $\frac{1}{2} \log \frac{1+z}{1-z}$

M. $e^{i \log z}$

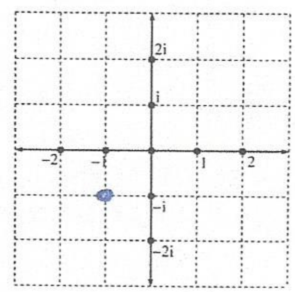
N. none of the above

21-25 Draw the following points or figure with as accurately as you can.

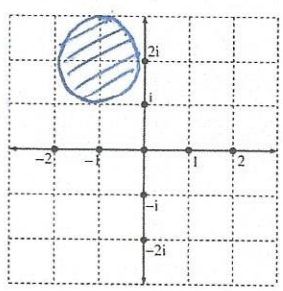
(21) Draw the point $z = 2 - 2i$.



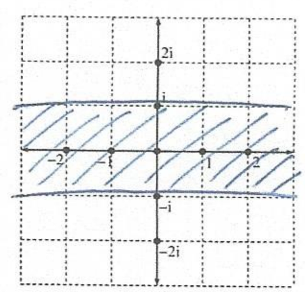
(22) Draw the point $\overline{iz}$, where $z = 1 + i$.



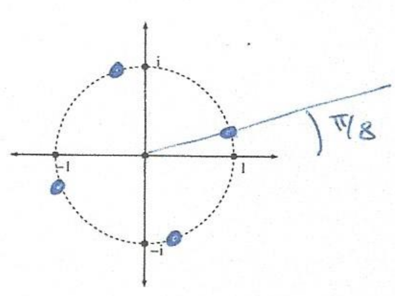
(23) Draw the region $|z + 1 - 2i| \leq 1$.



(24) Draw the region $|\mathrm{Im}(z)| \leq 1$.



(25) Draw all solutions of $z^4 = i$



21-30 TRUE/FALSE: Write T (for true) or F (for false) in each box.

(26) T The function e^z is entire.

(27) T If $f = u + iv$ is analytic and real valued, then f must be constant.

(28) F $|1 - z^2|$, attains a maximum value somewhere on the plane.

(29) T If f has an anti-derivative on domain D, then integral of f around any closed contour in D is zero.

(30) T If f is analytic on a disk D, then f must have an anti-derivative on D.

(31) T The function $\tan(z)$ is analytic on $\{z : |z| < 1\}$.

(32) F A polynomial of degree n must have n distinct zeros.

(33) F $f(x + iy) = 2xy + i(x^2 - y^2)$ is analytic on the plane.

(34) F Suppose $f = u + iv$. If the partials of u and v exist at a point z_0 and satisfy the Cauchy-Riemann equations at z_0, then f is differentiable at z_0 .

(35) F A function u is harmonic if $u_{xx} = u_{yy}$.

36-40: Give a precise statement of each definition or result.

(36) Define " f is analytic in an open set".

f is analytic on D if f is differentiable at every point of D.

(37) State the coincidence principle.

An analytic function f on a domain D is uniquely determined over D by its values in a domain, or along a line segment, contained in D.

(38) State Cauchy's formula.

If f is analytic everywhere inside and on a simple closed contour C taken in a positive sense and z_0 is inside C, then

$$f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0}\, dz$$

(39) State the Cauchy-Riemann equations for $f = u + iv$.

$$u_x = v_y$$
$$u_y = -v_x$$

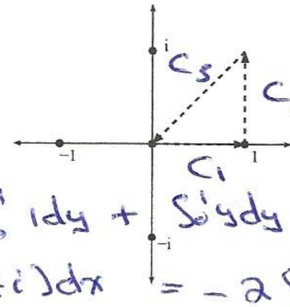
(40) Define simply connected domain.

D is simply connected if every simple closed contour encloses only points of D.

40-45 Evaluate each integral for the given contour; put your answer in the box.

$\int_{C_1} = \int_0^1 x\,dx = \frac{1}{2}$

(41) $\int_C \bar{z}dz$

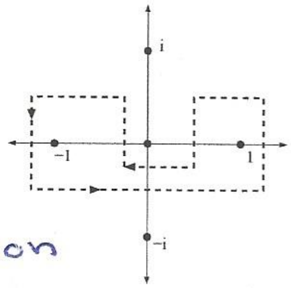


$\boxed{i}$

$\int_{C_2} = \int_0^1 (1-iy)\,i\,dy = i\int_0^1 1\,dy + \int_0^1 y\,dy = i + \frac{1}{2}$

$\int_{C_3} = -\int_{-C_3} (1-i)x(1+i)\,dx = -2\int_0^1 x\,dx = -1$

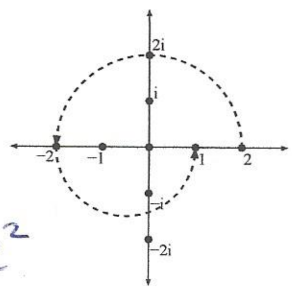
(42) $\int_C \sin(e^z)dz$



$\boxed{0}$

entire function

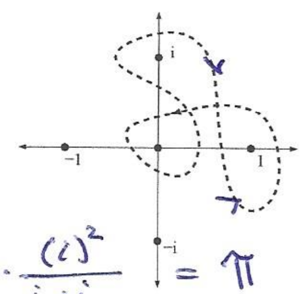
(43) $\int_C e^z dz$



$\boxed{e - e^2}$

$= \int_2^1 e^x dx = e^1 - e^2$

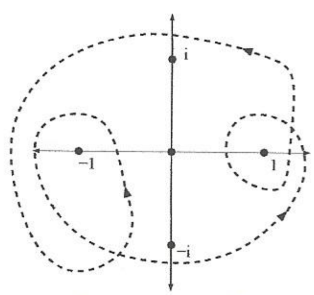
(44) $\int_C \frac{z^2}{z^2+1}dz$



$\boxed{\pi}$

$\int_C \frac{z^2\,dz}{(z-i)(z+i)} = -2\pi i \cdot \frac{(i)^2}{i+i} = \pi$

(45) $\int_C \frac{dz}{z^2-1}$



$\boxed{0}$

$= \int \frac{1}{2}\left(\frac{1}{z-1} - \frac{1}{z+1}\right) = \frac{1}{2}\cdot 2\cdot 2\pi i - \frac{1}{2}\cdot 2 \cdot 2\pi i$

46-50: Answer each question.

(46) Write the function $f(z) = z^3$ in the form $u(x, y) + iv(x, y)$.

$$\underbrace{(x^3 - 3xy^2)}_{u} + i\underbrace{(3x^2y - y^3)}_{v}$$

(47) Give an example of a function this is analytic on the whole plane except for the points $z = i$ and $z = -i$.

$$\frac{1}{(z+i)(z-i)}$$

(48) Given an example of an entire function that never equals 1.

$$e^z + 1$$

(49) Is the function $u(x, y) = x^2 + y^2$ harmonic? Explain why or why not.

$u_{xx} = 2, \quad u_{yy} = 2$

so $u_{xx} + u_{yy} = 4 \neq 0$

so u is not harmonic

(50) Evaluate $\int_C \exp(2z)z^{-4}dz$ where C is the postively oriented unit circle.

Let $f(z) = e^{2z}$

$$\int_C \frac{f(z)}{(z-0)^4}dz = \frac{2\pi i}{3!} f'''(0) = \frac{2\pi i}{6} 8 \cdot e^0$$

$$= \frac{8\pi i}{3}$$