

**MAT 331 Fall 2017, Practice Quiz 4**  
**Quiz 4 on Tuesday, October 31, 2017 (30 minutes)**

Name	ID	Score
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For answers that are real numbers, include all non-zero digits to the left of the decimal place, include the decimal place in a box, and as many digits to the right of the decimal place as will fit in the remaining boxes. Truncate, do not round, e.g., given five boxes for  $\sqrt{7} = 2.64575131106\dots$ , write “2.645”. If a number has no digits to the left of the decimal point, start with the decimal point, e.g., given ten boxes, write  $1/\sqrt{2}$  as “.707106781”. If a number is negative, use the leftmost box for the negative sign. Right justify integer answers, and place blanks (or zeros) in any remaining boxes on the left. For example, given 10 boxes to write  $2^{20}$  either write “0001048576” or “1048576” preceded by three blank boxes.

Problems (1)-(3) are about the following  $10 \times 10$  matrix:

$$A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{10} \\ \frac{1}{2} & 1 & \frac{1}{2} & \cdots & \frac{1}{9} \\ \frac{1}{3} & \frac{1}{2} & 1 & \cdots & \frac{1}{8} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{10} & \frac{1}{9} & \frac{1}{8} & \cdots & 1 \end{pmatrix}$$

(1) 

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Use the `toeplitz` command make  $A$  and find its determinate.

(2) 

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Solve  $Ay = x$  with  $x = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0)'$ . What is the first coordinate of  $y$ ?

(3) 

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For  $A$  in part (1), what is the largest eigenvalue of  $A$ ?

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Problems (4)-(6) are about the  $10 \times 10$  matrix (that depends on a real number  $t$ ):

$$B = \begin{pmatrix} 1 & t & 0 & \cdots & 0 & 0 & 0 \\ t & 1 & t & \cdots & 0 & 0 & 0 \\ 0 & t & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & t & 1 & t \\ 0 & 0 & 0 & \cdots & 0 & t & 1 \end{pmatrix}$$

(4) 

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Find the determinate of  $B$  when  $t = 1/2$ .

(5) 

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Solve  $By = x$  where  $t = 1/2$  and  $x = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)'$ . The answer is a binary vector; put a 0 or 1 in each box.

(6) 

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Let  $f(x) = \det(B(x))$ . Find the largest value of  $x \in [0, 1]$  where this is zero.

(7) 

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In the following graph, how many paths of length 20 start at vertex  $A$  and end at vertex  $B$ ?

