MAT 331 Fall 2017, Homework 0
Summing the digits of π

QUESTION:

(1) What is the sum of the first $N = 10,000$ digits of π? For example, the sum of the first three digits is $3 + 1 + 4 = 8$.

(2) If the digits of π are uniformly random in $\{0, 1, \ldots, 9\}$ what do we expect the sum to be? How far apart are the actual and expected sums?

(3) For $1 \leq k \leq N$, plot the difference between the expected and the actual sum of the first k digits of π. Do you see any pattern?

(4) Draw a histogram of how many times each digit is used. Which digit is used the most and which is used the least?

SOLUTION:

(1) The first step is to form a N-long vector whose entries are single digits integers correspodning to the digits of π. The code I used is

```
N=10000;
y=char(vpa(pi,N));
x(1)=str2num(y(1));
y=char(vpa(pi,N));
for k=2:N
   x(k)=str2num(y(k+1));
end
t=sum(x)
```

This creates a single character string `y` that is 10,001 characters long (there is an extra character for the decimal point). We then convert this to a string of integers `x`, remembering to skip the decimal place. The answer to the first part is `t= 44890` .

(2) If the digits were uniformly random in $\{0, 1, \ldots, 9\}$ then the average size of a digit would be $a = (0+1+\cdots+9)/10 = 4.5$, and the sum of $10,000$ such digits would be $45,000$. The difference between this and the actual sum is $45000 - 44890 = 110$.

(3) To compute the sum of the first k digits of π we can either use a loop

```
c(1)=x(1);
for k=2:N
   c(k)=c(k-1)+x(k);
end
```

or a built-in MATLAB command that does the same thing:

```
c=cumsum(x);
```

We want to plot the difference between this and $4.5k$:

```
figure;
hold on;
grid on;
title('The difference between acutal and expected  sums')
d=c-4.5*[1:N]
plot(d);
```

After the figure appeared on the screen I used the “File” and “Save As”buttons to save the figure as `pi_sum.pdf` in PDF format. The resulting picture is shown in Figure 1.

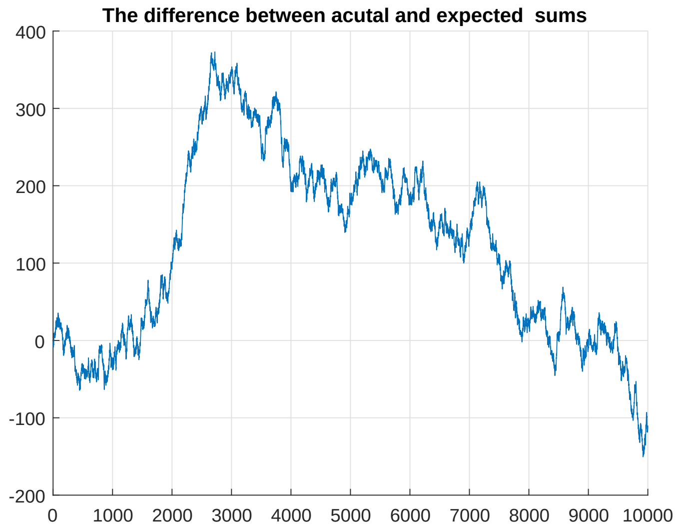

FIGURE 1. This plot the difference between the sum of the first k digits of π and the expected size of the sum, which is $(4.5)k$. At least to the naked eye, the difference seems random and no pattern in apparent.

(4) We plot the histgram using `hist(x,10)` which produces the figure shown on the left of Figure 2. The right side of the figure is an enlargement of the top of the histogram (made using the magnifing glass button on the top of the figure window). The most common digit is and the least common digit is 8.

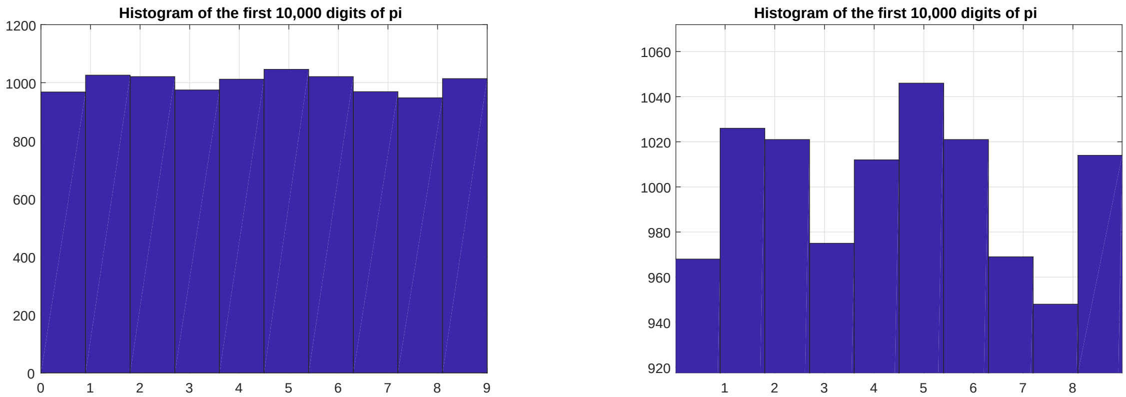

FIGURE 2. On the left is the histogram of the first 10,000 digits of π. On the right is an enlargement of the top of the histogram that shows that 5 is the most common digit and 8 is the least common.