(1) Does $\{\sin(nx)\}_1^\infty$ converge in the L^1 norm on $[0, 2\pi]$? Prove your answser.

(2) Give an example of a sequence of functions $\{f_n\}$ which converges to the constant zero function in L^1, but so that $f_n(x)$ does not converge to zero at any point of $[0, 1]$.

(3) If $f_n \to f$ in the L^1 norm, show that there is a subsequence f_{n_k} which converges a.e. to f.

(4) Give an example of a function which is in $L^2(\mathbb{R})$ but not in $L^p(\mathbb{R})$ for any $p \in [1, 2) \cup (2, \infty]$.

(5) Let $H = L^2([0, 1])$ and let $K \subset H$ be defined as $K = \{f \in H : \int_{[0,1]} f \, dm = 0\}$. Show that K is a closed vector subspace of H. Find the element of K that is closest to $g(x) = x$ (note $g \in H \setminus K$).