MAT 324, Fall 2015
PROBLEM SET 3, Due Thursday, September 24
Measurable sets

If $x \in [0,1]$ has a unique decimal expansion, let $d_n(x) \in \{0, 1, \ldots 9\}$ be the nth decimal digit of x. Otherwise let $d_n(x)$ be defined using the expansion of x then ends in an infinite string of 9's.

Prove that each of the following sets is measurable (in fact, they are all Borel).

(1) $X_1 = \{x \in [0,1] : \sum_{n=1}^{\infty} d_n(x) < \infty\}$.

(2) $X_2 = \{x \in [0,1] : d_1(x) \leq d_2(x) \leq \ldots\}$.

(3) $X_3 = \{x \in [0,1] : d_n(x) \neq 5 \text{ for all } n = 1, 2, 3, \ldots\}$.

(4) $X_4 = \{x \in [0,1] : \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} d_n(x) = 4\}$.

(5) $X_5 = \{t \in \mathbb{R} : \sum_{n=1}^{\infty} \sin(2^n t) \text{ converges}\}$.