

MAT 324, Fall 2015
PROBLEM SET 2, Due Tuesday, September 15
Measure Zero Sets

- (1) Show that the set of real numbers that do not have a unique decimal expansion has measure zero.
- (2) Let $E \subset [0, 1]$ be the set of real numbers between 0 and 1 whose decimal expansion contains no 7's. Show this set has measure zero.
- (3) Let $X = \{x \in [0, 2\pi] : |\sin(nx)| \leq \frac{1}{n} \text{ for all } n = 1, 2, 3, 4, \dots\}$. Show that X has measure zero.
- (4) Suppose $E \subset [0, 1]$ has measure zero and let $F = \{x^2 : x \in E\}$. Show F has measure zero.
- (5) Suppose $E \subset [0, 1]$ has measure zero and suppose $f : [0, 1] \rightarrow [0, 1]$ is continuous. Must $f(E) = \{f(x) : x \in E\}$ be measure zero? Give a proof or a counterexample.