MAT 324, Fall 2015, Midterm Review
Midterm is Thursday, Oct 15 (usual time and room)

Exam format:
- 8 definitions and statements (40 points)
- 4 examples (20 points)
- prove 2 propositions from list below (20 points)
- do 2 problems (20 points)

Know the following terms and results: Countable, uncountable, open, closed, continuous, Riemann integral, Riemann's criterion, fundamental theorem of calculus, Cauchy sequence, complete metric space, null set, Cantor set, Cantor singular function, outer measure, additivity, sub-additivity, Lebesgue measurable set, Lebesgue measure, sigma field, symmetric difference, Bofel set, complete measure, completion of a measure, Lebesgue measureable function, Borel function, essential supremum, simple function, Lebesgue integral, Fatou's lemma, monotone convergence theorem, integrable function, dominated convergence theorem, Beppo-Levi theorem, characterization of Riemann integrable functions, Riemann-Lebesgue lemma.

Know various examples from class and text including: non-measurable sets, Cantor set and function, various sequences where limit of integrals is not the integral of the limiting function, examples of measurable sets and functions, . . .

Be able to prove:
Prop 2.16: if A is measurable and $m(A\Delta B) = 0$ then B is measurable and $m(A) = m(B)$.
Prop 2.27: The completion of a σ-field $\mathcal{F}$ with respect to a measure μ is $\{G \cup N : G \in \mathcal{F}, N \subset F \in \mathcal{F} \text{ with } \mu(F) = 0\}$.
Prop 3.15: If f, g are measurable $\text{esssup}(f + g) \leq \text{esssup}(f) + \text{esssup}(g)$.
Prop 4.5: for simple functions, the two definitions of ingtegral agree.
Prop 4.10: f is measurable iff both f^+ and f^- are measurable.
Prop 4.15: For any measureable $f \geq 0$, there is a sequence of simple functions $\{s_n\}$ with $s_n \nearrow f$ everywhere.
Prop 4.23: Assume that f is mesurable and $f \geq 0$. Then $\int f dm = 0$ iff $f = 0$ a.e..
Prop 4.29: For a sequence of non-negative measurable functions $\{f_n\}$, we have

$$\int (\sum_{n=1}^{\infty} f_n) dm = \sum_{n=1}^{\infty} \int f_n dm.$$

Be able to do all homework problems; problems on midterm will be similar (but not necessarily identical). Be prepared to show certain sets and functions are measurable, and compute integrals of certain functions.