MAT 324, Fall 2015, Final Exam Review
Final Exam is Friday, Dec 11, 11:15am-1:45pm
in Melville Library W4530 (usual room)

Exam format:
- 5 definitions and statements (25 points)
- 5 examples (25 points)
- prove 2 propositions from list below (20 points)
- do 3 problems out of 5 (30 points)

In addition to the terms and results listed on the Midterm Review sheet, know the following: L^1, L^2, L^∞, L^p, norm, Cauchy sequence, complete metric space, Banach space, inner product, Hilbert space, Schwarz inequality, parallelogram law, polarization identity, orthogonal vectors, orthogonal projection, Hölder's inequality, Minkowski's inequality, product σ-field, monotone class, monotone class theorem, Fubini's theorem, absolute continuity of measures, mutual singularity of measures, Radon-Nikodym theorem, σ-finite measure space, Lebesgue decomposition, Lebesgue-Stieltjes measures, distribution function, absolutely continuous functions, bounded variation, signed measure, Hahn-Jordan decomposition, Borel-Cantelli lemma

Know various examples from class and text including (but not limited to): functions in one L^p space but not in another, sequence that converge pointwise a.e. but not in L^1 (and conversely), functions of two variables whose iterated integrals are not equal, functions that are absolutely continuous, bounded variation (and non-examples).

Be able to prove the following propositions from the text:

Prop 5.7: If μ is a finite measure then $L^2(\mu) \subset L^1(\mu)$.

Prop 5.12(i): The parallelogram law.

Prop 7.2: Let ν, μ be finite measures on the measure space $(\Omega, \mathcal{F})$. Then $\nu \ll \mu$ if and only if for every $\epsilon > 0$ there exists a $\delta > 0$ such that for $F \in \mathcal{F}$, $\mu(F) < \delta$ implies $\nu(F) < \epsilon$.

Prop 7.11(i): If $\mu, \lambda_1, \lambda_2$ are measures on a σ-field $\mathcal{F}$, then if $\lambda_1 \perp \mu$ and $\lambda_2 \perp \mu$, then $(\lambda_1 + \lambda_2) \perp \mu$.

Prop 7.15: If $F : \mathbb{R} \to \mathbb{R}$ is monotone increasing and the left and right limits of F exist at every point $x \in \mathbb{R}$, then F has at most countably many points of discontinuity.

Prop 7.21: If $f \in L^1([a, b])$, then the function $F(x) = \int_[a, x] f dm$ is absolutely continuous.

Prop 7.27(i): If F is absolutely continuous on $[a, b]$, then F is bounded variation on $[a, b]$.

You will be asked to do three out of five problems from a list taken from the problem sets and the following problems:

1. Prove that $L^1(\mathbb{R}, dm)$ is not a Hilbert space with its usual norm.

2. Let
$$E_\alpha = \{x \in \mathbb{R} : |x - \frac{p}{q}| < q^{-\alpha} \text{ for infinitely many integers } p, q\}.$$
Show that E_α has Lebesgue measure zero if $\alpha > 2$.

3. If f is absolutely continuous on $[0, 1]$ and $E \subset [0, 1]$ has Lebesgue measure zero, prove that $f(E)$ also has Lebesgue measure zero. Deduce that the image of a Lebesgue measurable set under an absolutely continuous function is also Lebesgue measurable.