MAT 324, Fall 2006
PROBLEM SET 1

1. Let X be the set of all real roots of all polynomials with integer coefficients. Is X countable or uncountable? Explain why.

2. Define $x \sim y$ if $x - y$ is rational. Prove this is an equivalence relation on the reals. How many distinct equvalence classes are there?

3. Is the function $f(x) = \sum_{n=0}^{\infty} 2^{-n} \sin(2^n x)$ Riemann integrable on $[0, 2\pi]$? Explain why or why not.

4. Is there a compact, uncountable set of real numbers which contains no rational numbers? Give an example or prove no such set exists.

5. What is the average distance between two random points in $[0, 1]$? We have not had enough theory yet to make this precise, but see if you can come up with a plausible number and explaination for it.