MAT 320 Fall 2021, Sample Final Exam Real final is 8:00-10:45am Tuesday, December 14, 2021

Name	ID	Section

1-20	21	22	23	24	total

THIS MIDTERM IS WORTH 100 POINTS. THERE ARE 20 TRUE/FALSE QUESTIONS WORTH 2 POINTS EACH, AND 4 PROOFS WORTH 15 POINTS EACH (24 QUESTIONS TOTAL). NO BOOKS OR NOTES ARE ALLOWED. THERE ARE FIVE PRINTED PAGES AND TWO BLANK PAGES.

(1)-(20) TRUE/FALSE: put a T or F in each box.

(1) ☐ A continuous function on $\mathbb{R}$ that is unbounded above and below takes on all real values.

(2) ☐ An increasing function on $\mathbb{R}$ is continuous except on a set of length zero.

(3) ☐ A continuous function on $\mathbb{R}$ can only take the value 0 countable many times.

(4) ☐ If f is differentiable everywhere on $[a, b]$ and $f'(x) > 0$ everywhere, then f is strictly increasing on $[a, b]$.

(5) ☐ If f is differentiable everywhere on $[a, b]$ then f' must be bounded on $[a, b]$.

(6) ☐ Suppose $f(x) = 1$ if x is in the Cantor middle third set $E \subset [0, 1]$, and $f(x) = 0$ otherwise. Then f is Riemann integrable on $[0, 1]$.

(7) ☐ If $1 \geq f_1(x) \geq f_2(x) \geq f_3(x) \geq \dots$ and $f_n(x) \to 0$ for every $x \in [0, 1]$ then $\int_0^1 f_n \to 0$.

(8) ☐ The sequence $f_n(x) = \sin(nx)$ converges uniformly to some limit on $[0, 2\pi]$.

(9) If $\{f_n\}$ are 1-Lipschitz functions that converge uniformly on $[a, b]$ to f, then f is also 1-Lipschitz.

(10) If (f_n) are continuous and positive on $[0, 1]$ and if $\sum f_n(x)$ converges for every $x \in [0, 1]$ then $f(x) = \sum f_n(x)$ is continuous.

(11) If (f_n) is a sequence of positive, continuous functions on $[0, 1]$ so that $\int_0^1 f_n \to 0$, then $f_n(x) \to 0$ for every $x \in [0, 1]$.

(12) If $\sum x_n$ converges, then $\sum(-1)^n x_n$ converges.

(13) If $\sum y_n$ converges, then $\sum y_n^2$ also converges.

(14) $f(x) = \sum_{n=1}^{\infty} n^{-1} \cos(nx)$ converges for every $x \in \mathbb{R}$.

(15) A set can be both open and closed.

(16) If $f : \mathbb{R} \to \mathbb{R}$ is continuous then $\{x : f(x) > 0\}$ is an open set.

(17) The union of two compact sets is also compact.

(18) If $A \subset \mathbb{R}$ is closed and f is continuous on $\mathbb{R}$, then $f(A)$ is also closed.

(19) $d(x, y) = |x - y|^2$ is a metric on $\mathbb{R}$.

(20) $d(f, g) = \int_0^1 |f - g|$ is a metric on $\mathcal{R}[0, 1]$ (Riemann integrable functions).

In problems 21-24, you may use results from class/textbook if you quote them correctly.

(21) If K is a compact subset of a metric space (S, d), show that K is closed and bounded. Bounded means that $K \subset \{x \in S : d(x, x_0) \leq B\}$ for some $x_0 \in S$ and $B < \infty$.

(22) If $E \subset \mathbb{R}$ is a closed set, show that $d(x) = \inf\{|x - y| : y \in E\}$ is a continuous function on $\mathbb{R}$ so that $E = \{x : d(x) = 0\}$.

(23) If the partial sums (s_n) of $\sum a_n$ are bounded prove that $\sum a_n/n$ is convergent.

(24) Prove that the set of rationals numbers is not the countable intersection of open sets.