MAT 320 Fall 2021, Sample Midterm 2.
Real Midterm is Thursday, November 11, 2021

Name	ID	Section

1-20	21	22	23	24	total

THIS MIDTERM IS WORTH 100 POINTS. THERE ARE 20 TRUE/FALSE QUESTIONS WORTH 2 POINTS EACH, AND 3 PROOFS WORTH 20 POINTS EACH (23 QUESTIONS TOTAL). NO BOOKS OR NOTES ARE ALLOWED.

(1)-(20) TRUE/FALSE: put a T or F in each box.

(1) ☐ $\lim_{x\to 0} x^2 \sin(1/x)$ exists.

(2) ☐ $\lim_{x\to 0} \exp(1/x)$ exists.

(3) ☐ If $\lim_{x\to c} f(x)$ exists, then $\lim_{x\to c} f^2(x)$ also exists.

(4) ☐ If $\lim_{x\to c} f^2(x)$ exists, then $\lim_{x\to c} f(x)$ also exists.

(5) ☐ There is a function $f : \mathbb{R} \to \mathbb{R}$ that is discontinuous on $\mathbb{Z}$ and continuous everywhere else.

(6) ☐ There is a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous on $\mathbb{Z}$ and discontinuous everywhere else.

(7) ☐ If f is continuous on $[a, b]$ then f is bounded on $[a, b]$.

(8) ☐ A function $f : \mathbb{R} \to \mathbb{R}$ is continuous at 0 iff $f(0) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$.

(9) $\square$ If $f : [0,1] \to \mathbb{R}$ is continuous then f has an absolute maximum.

(10) $\square$ If $f : A \to \mathbb{R}$ has an absolute maximum, then it has an absolute minimum.

(11) $\square$ If f is monotone on an interval I, then $J = f(I)$ is also an interval.

(12) $\square$ If f is differentiable everywhere on $[a,b]$ then f' is continuous on $[a,b]$.

(13) $\square$ If $f : (a,b) \to \mathbb{R}$ is continuous, then it is uniformly continuous.

(14) $\square$ If f is not differentiable at c then f cannot take an absolute maximum at c.

(15) $\square$ If $f : [a,b] \to \mathbb{R}$ is differentiable at every point of $[a,b]$ and $f(a) < f(b)$ then $f'(c) > 0$ for some $c \in (a,b)$.

(16) $\square$ If $f : \mathbb{R} \to \mathbb{R}$ is odd and differentiable everywhere, then $f'(-1) = -f'(1)$.

(17) $\square$ If f is Riemann integrable on $[a,b]$ then so is $|f|$.

(18) $\square$ Any increasing function f on $[a,b]$ is Riemann integrable.

(19) $\square$ If $f : [0,1] \to \mathbb{R}$ is Riemann integrable on $[\frac{1}{n}, 1]$ for every n, then f is Riemann integrable on $[0,1]$.

(20) $\square$ If f is Riemann integrable on $[a,b]$ and $g = f$ except at countably many points, then g is also Riemann integrable on $[a,b]$.

(21) Suppose $f : (0, 1) \to \mathbb{R}$ is bounded but $\lim_{x\to 0} f(x)$ does not exist. Show there are sequences $(x_n), (y_n)$ in $(0, 1)$ that both converge to zero, but such that $(f(x_n))$ and $(f(y_n))$ converge to different limits.

(22) Suppose $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b). If $\lim_{x\to a} f'(x) = A$ show that f is differentiable at a and $f'(a) = A$.

(23) Given an example of two Riemann integrable functions f, g on $[0, 1]$ so that the composition $f \circ g$ is not Riemann integrable. (Hint: use some of the discontinuous functions discussed in class and the textbook.)