SAMPLE MIDTERM 1, MAT 319 & 320 Fall 2021
Real midterm is in lecture, Thursday, September 30, 2021

Name	ID	Section

1-20	21	22	23	24	total

THIS MIDTERM IS WORTH 100 POINTS. THERE ARE 20 TRUE/FALSE QUESTIONS WORTH 2 POINTS EACH, AND 4 PROOFS WORTH 15 POINTS EACH (24 QUESTIONS TOTAL). NO BOOKS, NOTES OR CALCULATORS ARE ALLOWED. THERE ARE FOUR PRINTED PAGES.

TRUE/FALSE: put a T or F in each box.

(1) ☐ If $A = \{1, 2, 6, 8, 10\}$, $B = \{2, 3, 5, 6, 7, 9\}$, then $A \cap B = \{2, 6\}$.

(2) ☐ $\{x \in \mathbb{Z} : x < 100\}$ is a finite set.

(3) ☐ If A, B are both infinite sets, then $A \cup B$ is infinite.

(4) ☐ $f(n) = n^3$ is a bijective map from $\mathbb{Z}$ to itself.

(5) ☐ If $f : X \to Y$ and $g : Y \to Z$ are injective, then $g \circ f : X \to Z$ is injective

(6) ☐ There is a bijection between $\mathbb{N}$ and $\mathbb{Z}$.

(7) ☐ A subset of a countable set must also be countable.

(8) ☐ There is a bijective map from the interval $(0, 1)$ to $\mathbb{R}$.

(9) Every interval has a supremum.

(10) The infimum and supremum of an infinite, bounded set must be different.

(11) If a is irrational, then a^2 is irrational.

(12) $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$.

(13) $|x - 10| < 4$ implies $x > 0$.

(14) Every monotone sequence has a limit.

(15) If $\{a_n\}$ converges then $\{a_{n^2}\}$ also converges.

(16) $\{\sin(n^2)\}_1^\infty$ has a convergent subsequence.

(17) If $\{x_n\}_0^\infty$ satisfies $x_{n+1} = x_n + \exp(x_{n-1})$, for $n \in \mathbb{N}$, then $\{x_n\}$ is increasing.

(18) $1 = \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n} + \ldots$.

(19) $\sum_{n=1}^\infty (-1)^n/\sqrt{n}$ converges.

(20) $\sum_{n=1}^\infty \frac{1}{n}$ converges.

(21) State the Density Theorem and use it to prove that for any $x \in \mathbb{R}$ and $\epsilon > 0$ there are rational numbers a, b so that $x \in (a, b)$ and $|a - b| < \epsilon$.

(22) Let $\mathcal{P}$ be all polynomials (of any degree) $p(x) = a_0 + a_1x + \cdots + a_nx^n$ that have integer coefficients. Is $\mathcal{P}$ countable or uncountable? Prove your answer.

(23) Let $A \subset \mathbb{R}$ be infinite and bounded above. Show there is an increasing sequence in A whose limit is $\sup A$.

(24) Suppose that $0 < a_0 < 1$ and $a_{n+1} = a_n(1 - a_n)$ for $n = 0, 1, 2, 3, \ldots$. Prove that $\{a_n\}$ is a bounded sequence.