MIDTERM 2
MAT 141
11/17/00

Name	Sec.

ID number

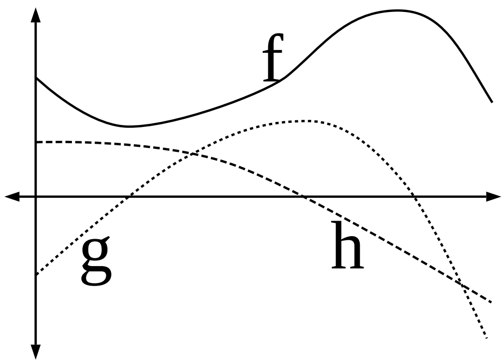
1	2	3	4	total

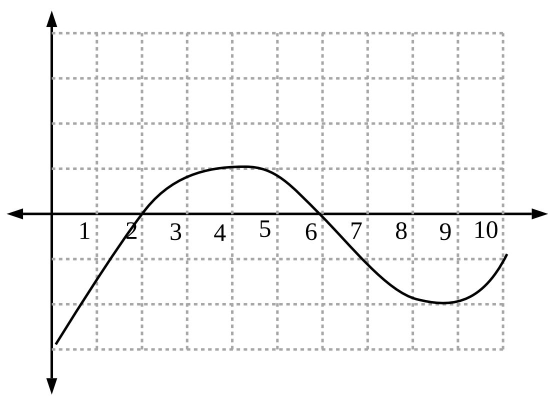
THIS EXAM IS WORTH 40 POINTS. PUT ALL ANSWERS IN THE SPACE PROVIDED. YOU MAY USE THE BACKS OF PAGES FOR SCRATCH WORK.

1. **(1 pt each, 20 pts total)** Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

   (i) ☐ Suppose $f(x) = x^4 + x^3 + 1$. Then f has
   (a) an absolute maximum at $x = 0$ **(b)** an absolute minimum at $x = 0$ **(c)** a local minimum at $x = 3/4$ **(d)** an absolute minimum at $x = -3/4$ **(e)** a local minumum at $x = 0$ **(f)** none of these.

   (ii) ☐ Suppose f, g, h are graphed on the left below. Which of the following is a possible relationship based on the graphs?
   (a) $g = f'$, $h = g'$ **(b)** $g = h'$, $f = g'$ **(c)** $f = g'$, $h = f'$ **(d)** $f = h'$, $g = f'$ **(e)** $h = f'$, $g = h'$ **(f)** $h = g'$, $f = h'$





   (iii) ☐ Suppose f is graphed on the right above. If we use Newton's method with initial guess $x_0 = 3$, then our next guess with be approximately $x_1 =$
   (a) 9 **(b)** 7 **(c)** 6 **(d)** 4 **(e)** 3 **(f)** 1

   (iv) ☐ Suppose f has values given by the following table. What is the derivative of $h(x) = f^2(2x)$ at $x = 2$?

x	0	1	2	3	4
$f(x)$	2	3	5	4	4
$f'(x)$	0	2	-1	-1	1

   (a) -4 **(b)** 8 **(c)** 12 **(d)** 16 **(e)** 20 **(f)** none of these.

(v) What is the slope of the curve given by $x^2 - xy + y^2 = 7$ at the point $(x, y) = (-1, 2)$?
(a) 1 **(b)** $\frac{4}{5}$ **(c)** $\frac{3}{4}$ **(d)** 0 **(e)** $\frac{6}{5}$ **(f)** none of these.

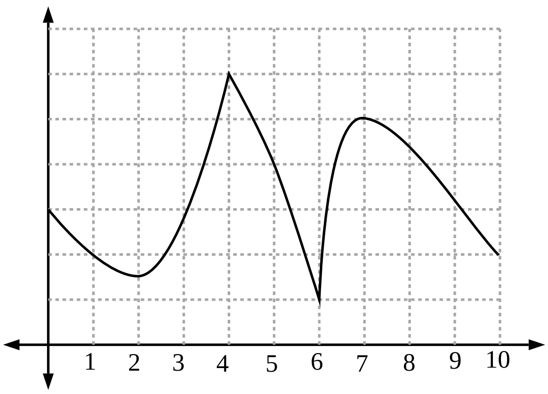
(vi) Find $\frac{dy}{dx}$ if $x^2y + y = 1$
(a) $\frac{dy}{dx} = \frac{-2xy}{x^2+1}$ **(b)** $\frac{dy}{dx} = \frac{2xy}{x^2-1}$ **(c)** $\frac{dy}{dx} = \frac{xy}{2y+x^2}$ **(d)** $\frac{dy}{dx} = \frac{-2xy}{y+x}$ **(e)** $\frac{dy}{dx} = (1 - 2xy)(x^2 - 1)$ **(f)** none of these.

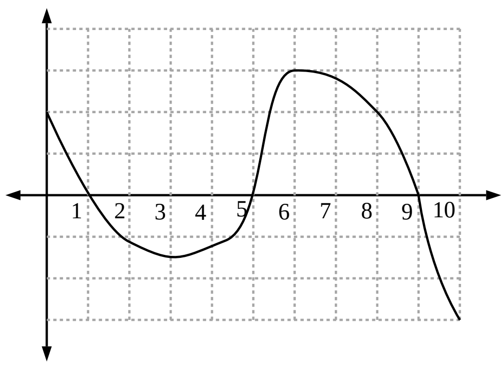
(vii) Suppose we try to find roots of $x^3 - x - 2 = 0$ by Newton's method using the intial guess $x_0 = 1$. The next guess with be
(a) $x_1 = 0$ **(b)** $x_1 = 1.5$ **(c)** $x_1 = 2$ **(d)** $x_1 = 3$ **(e)** $x_1 = -1$ **(f)** none of these.

(viii) Suppose $f(x) = \frac{1}{x} - 3$. Then the recursion formula in Newton's method is
(a) $x_{n+1} = x_n - 3$ **(b)** $x_{n+1} = 2x_n - 3x_n^2$ **(c)** $x_{n+1} = x_n + 3x_n^2$ **(d)** $x_{n+1} = x_n - x_n^2$ **(e)** $x_{n+1} = x_n - 3/x_n^2$ **(f)** none of these.

(ix) Suppose $f'(x) = x^2 + \sin^2(x)$. Then on $(-\infty, \infty)$ f is
(a) concave up **(b)** concave down **(c)** increasing **(d)** decreasing **(e)** constant **(f)** none of these.

(x) Suppose f is as graphed on the left below. The set of critical points of f is
(a) $\{2, 4\}$ **(b)** $\{2, 7\}$ **(c)** $\{4, 6\}$ **(d)** $\{2, 4, 6, 7\}$ **(e)** $\{0, 2, 4, 6, 7, 10\}$ **(f)** none of these.





(xi) Suppose g' is graphed on the right above. Then g is increasing and concave up on the interval
(a) $[0, 1]$ **(b)** $[1, 3]$ **(c)** $[3, 5]$ **(d)** $[5, 6]$ **(e)** $[6, 9]$ **(f)** none of these.

(xii) Suppose g' is graphed on ther right above. The local maximum(s) of g (excluding endpoints) are exactly
(a) $1, 9$ **(b)** 5 **(c)** 3 **(d)** 1 **(e)** 6 **(f)** none of these.

(xiii) Suppose g' is graphed on the right above. The inflection points of g are exactly
(a) $1, 9$ **(b)** 1 **(c)** $3, 6$ **(d)** 5 **(e)** $1, 5, 9$ **(f)** none of these.

(xiv) $\square$ Find the linearization of $f(x) = x^2 + x$ at $x = 1$.
(a) $L(x) = 3x$ **(b)** $L(x) = (x-1)$ **(c)** $L(x) = -2(x-1)+1$ **(d)** $L(x) = 2x+1$ **(e)** $L(x) = 3(x-1)+2$ **(f)** none of these.

(xv) $\square$ Use differentials to estimate the change in the volume of a cube $S = x^3$ when the edge length goes from x_0 to $x_0 + dx$
(a) $3x_0^2dx$ **(b)** $6x_0dx$ **(c)** $12x_0^2dx$ **(d)** $3dx$ **(e)** $3x_0dx$ **(f)** none of these.

(xvi) $\square$ The solution of the inital value problem $\frac{dy}{dx} = \cos(x) + 1$, $y(\pi) = 0$ is
(a) $y = \sin(x) + 1$ **(b)** $y = \sin(x) + x$ **(c)** $y = \sin(x) + x - \pi$ **(d)** $y = \sin(x) + x + \pi$ **(e)** $y = \sin(x) + \pi$ **(f)** none of these.

(xvii) $\square$ Suppose $f'(x) = 1 - \sin^{10}(x)$. Then on the interval $[0, \frac{1}{2}\pi]$ the function f is
(a) increasing and concave down **(b)** increasing and concave up **(c)** decreasing and concave down **(d)** decreasing and concave up **(e)** constant **(f)** none of these.

(xviii) $\square$ What is the name of the following result: "Suppose f is continuous on $[a, b]$, differentiable on (a, b) and $f(a) = f(b) = 0$. Then there is a point $c \in (a, b)$ such that $f'(c) = 0$."
(a) The intermediate value theorem **(b)** Green's theorem **(c)** Rolle's theorem **(d)** The mean value theorem **(e)** The min-max theorem **(f)** none of these.

(xix) $\square$ The function $f(x) = x^3 + 3x^2 - 3x + 1$ has a point of inflection at $x =$?
(a) -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 2 **(f)** none of these.

(xx) $\square$ Use the linearization of $x^{1/3}$ at $x = 27$ to approximate $29^{1/3}$.
(a) $3\frac{1}{3}$ **(b)** $3\frac{1}{2}$ **(c)** $3\frac{7}{12}$ **(d)** $3\frac{2}{27}$ **(e)** $3\frac{5}{6}$ **(f)** none of these.

2. **(2 pts each, 10 pts total)** Find each of the following indefinite integrals

(i) $\int(x^3 - x^2 + 2)dx$, $\square$

(ii) $\int \sec^2(x)dx$,

(iii) $\int (3x+2)^9 dx$,

(iv) $\int \sin^4(t)\cos(t)dt$,

(v) $\int 6\sin^2(t^2)\cos(t^2)tdtdt$,

3. **(5 pts)** The coordinates of a particle in the xy-plane are differentiable functions of time t satisfying $dx/dt = -1m/sec$ and $dy/dt = 5m/sec$. How fast is the particle's distance to the origin changing as it passes through the point $(x, y) = (5, 12)$?

4. **(5 pts)** Your company can manufacture x hundred grade A tires and y hundred grade B tires a day where $0 \leq x \leq 4$ and

$$y = \frac{42 - 10x}{5 - x}.$$

Your profit on grade A tires is twice the profit on grade B tires. Find the most profitable number of each kind of tire to make.